From Reflection Data to Transmission Coda

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Abstract

Using the reciprocity theorem of the correlation type an implicit relation between seismic reflection and transmission data has previously been derived by Wapenaar et al. (2002). In this paper this relation is used to calculate transmission coda from reflection data measured at the surface. It is assumed that the transmission response can be represented by a generalized propagator consisting of a primary propagator and a coda operator. Using this representation it is possible to solve the implicit relation for the coda operator, using an eigenvalue decomposition on the correlation of the reflection response. The calculated coda response may be used in seismic reflection imaging to obtain an image in which the internal multiple scattering effects are suppressed. A simple example for the estimation of the transmission coda illustrates the discussed procedure.

Introduction

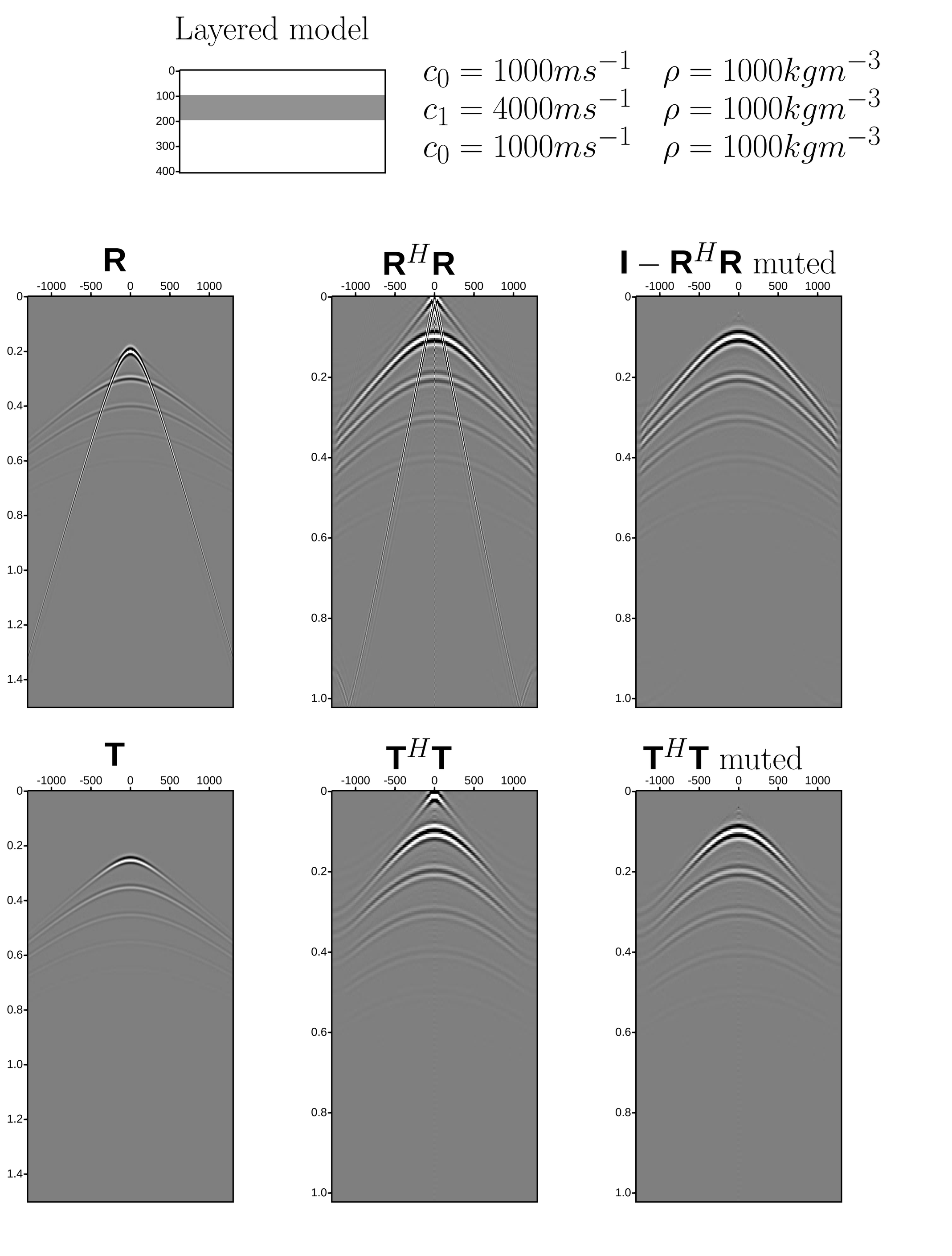


FIGURE 1: *The correlation of reflection and transmission data illustrates the principle of conservation of energy flux. The reflection energy ($\mathbf{R}^H\mathbf{R}$) together with the transmission energy ($\mathbf{T}^H\mathbf{T}$) is equal to the energy of the source ($\mathbf{I}$). This relation forms the basis of the derivation of transmission coda from correlated reflection data.*

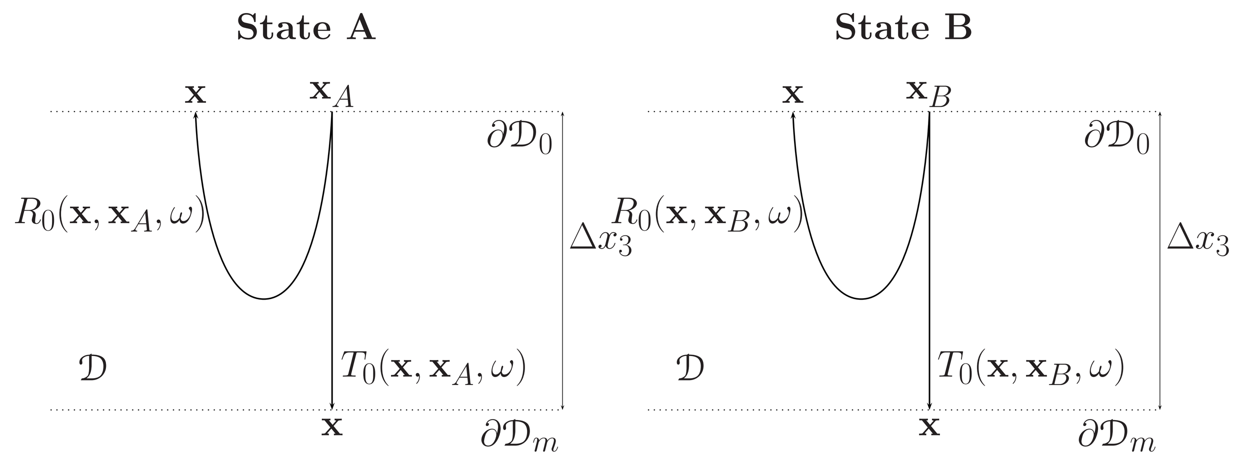
The one-way reciprocity relation of the correlation type in the frequency domain, introduced by Wapenaar and Grimbergen (1996):

$$\int_{\partial\mathbb{D}_0}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}d^2\mathbf{x}_H = \int_{\partial\mathbb{D}_m}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}d^2\mathbf{x}_H \quad (1)$$

is the starting point to derive a formal relation between reflection and transmission data. The medium parameters in both states A and B are assumed to be identical, lossless and 3-D inhomogeneous and the domain $\mathbb{D}$ is source free. $\partial\mathbb{D}_0$ and $\partial\mathbb{D}_m$ are two horizontal boundaries at $x_{3,0}$ and $x_{3,m}$, respectively, enclosing the domain $\mathbb{D}$ (see Figure 2). $\mathbf{x}_H$ denotes the horizontal coordinates (x_1, x_2) and * denotes complex conjugation. Both states, and the related flux-normalized up- and down-going fields P^- and P^+ respectively, are shown in the table of Figure 2. Substituting the two states in the table into equation (1) results in:

$$\delta(\mathbf{x}_{H,A}-\mathbf{x}_{H,B}) - \int_{\partial\mathbb{D}_0} R_0^*(\mathbf{x},\mathbf{x}_A)R_0(\mathbf{x},\mathbf{x}_B)d^2\mathbf{x}_H = \int_{\partial\mathbb{D}_m} T_0^*(\mathbf{x},\mathbf{x}_A)T_0(\mathbf{x},\mathbf{x}_B)d^2\mathbf{x}_H. \quad (2)$$

Here $R_0(\mathbf{x},\mathbf{x}_A)$ is the reflection response of the inhomogeneous medium in $\mathbb{D}$, including all internal multiples, for a source at $\mathbf{x}_A = (\mathbf{x}_{H,A}, x_{3,0})$ and a receiver at $\mathbf{x}$. $T_0(\mathbf{x},\mathbf{x}_A)$ is the transmission response of the inhomogeneous medium in $\mathbb{D}$. The subscript $_0$ denotes that no free surface multiples are included. A more intuitive interpretation of equation (1) is shown in Figure 1, where the conservation of energy is illustrated in pictures.



Field	State A	State B
Surface $\partial\mathbb{D}_0$		
P^+	$\delta(\mathbf{x}_H-\mathbf{x}_{H,A})s_A(\omega)$	$\delta(\mathbf{x}_H-\mathbf{x}_{H,B})s_B(\omega)$
P^-	$R_0(\mathbf{x},\mathbf{x}_A,\omega)s_A(\omega)$	$R_0(\mathbf{x},\mathbf{x}_B,\omega)s_B(\omega)$
Surface $\partial\mathbb{D}_m$		
P^+	$T_0(\mathbf{x},\mathbf{x}_A,\omega)s_A(\omega)$	$T_0(\mathbf{x},\mathbf{x}_B,\omega)s_B(\omega)$
P^-	0	0

FIGURE 2: *At $\mathbf{x}_A$ (state A) or $\mathbf{x}_B$, just above $\partial\mathbb{D}_0$, there is a source for downgoing waves. The half-spaces above $\partial\mathbb{D}_0$ and below $\partial\mathbb{D}_m$ are homogeneous. The half-space below $\partial\mathbb{D}_m$ is source free. The wave fields of both states are shown in the table.*

From equation (1) it is clear that there is no unique way to solve the full transmission response. However, by using a suitable representation of the transmission operator $\mathbf{T}$ there is a way to compute the transmission coda.

Transmission coda calculated from eigenvalues of correlated reflection data

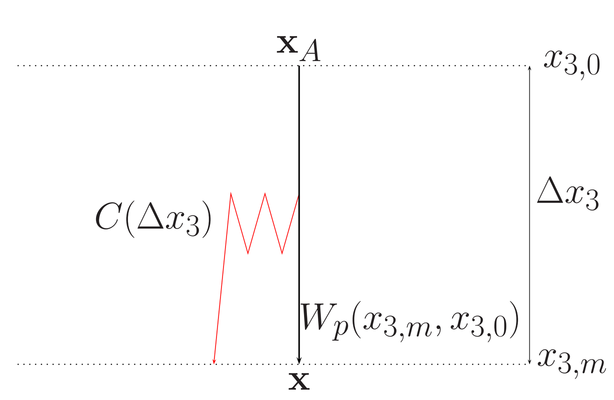


FIGURE 3: *The transmission operator is represented by a generalized primary propagator which consists of two parts: the primary propagator W_p for direct propagation and a coda operator C representing the internal multiples.*

For the current discussion we only need to know that the transmission response is written as a flux normalized generalized primary propagator W_g in the $\omega - p$ domain:

$$\begin{aligned} T_0(p,\mathbf{x}_A,x_{3,m},x_{3,0}) &= W_g(p,\mathbf{x}_A,x_{3,m},x_{3,0}) \\ T_0(p,\mathbf{x}_A,x_{3,m},x_{3,0}) &= W_p(p,\mathbf{x}_A,x_{3,m},x_{3,0})C(p,\mathbf{x}_A,\Delta x_3) \end{aligned} \quad (3)$$

where W_p is the primary propagator for downgoing waves between depth levels $x_{3,0}$ and $x_{3,m}$ and where C accounts for the coda due to multiple scattering caused by the inhomogeneities between the two depth levels and $\Delta x_3 = x_{3,m} - x_{3,0}$. The primary propagator is given by:

$$W_p(p,\mathbf{x}_A,x_{3,m},z_0) = \exp(-j\omega\int_{x_{3,0}}^{x_{3,m}}(c^{-2}(z)-p^2)^{\frac{1}{2}}dx_3)$$

The inverse of W_p can be approximated by W_p^* for propagating waves and gives the following relation

$$W_p^*(p,\mathbf{x}_A,x_{3,m},x_{3,0})W_p(p,\mathbf{x}_A,x_{3,m},x_{3,0}) \approx 1.0.$$

Furthermore, O'Doherty and Anstey (1971b) showed that for 1D media the coda operator $C(\omega)$ is related to the exponent of the power spectrum of the reflection coefficient:

$$C(p,\mathbf{x}_A,\Delta x_3) = \exp(-\mathcal{A}(p)\Delta x_3). \quad (4)$$

Matrix notation

Integral equation (1) is written in matrix notation to take into account the discrete measurements of seismic data in time and space and to be able to handle it in a computer.

$$\int_{\partial\mathbb{D}_0} R_0^*(\mathbf{x},\mathbf{x}_A)R_0(\mathbf{x},\mathbf{x}_B)d^2\mathbf{x}_H \equiv \boldsymbol{R}_0^H(\mathbf{x},\mathbf{x}_A)\boldsymbol{R}_0(\mathbf{x},\mathbf{x}_B)$$

$$= \left(R_0^*(\mathbf{x}_0,\mathbf{x}_A)\ldots R_0^*(\mathbf{x}_i,\mathbf{x}_A)\ldots R_0^*(\mathbf{x}_N,\mathbf{x}_A)\right)\begin{pmatrix} R_0(\mathbf{x}_0,\mathbf{x}_A) \\ \vdots \\ R_0(\mathbf{x}_i,\mathbf{x}_A) \\ \vdots \\ R_0(\mathbf{x}_N,\mathbf{x}_A)\end{pmatrix}$$

represents the correlation of two seismic shot record. Including multiple shot positions with a fixed spread acquisition geometry gives for the full matrix $\boldsymbol{R}$,

$$\mathbf{R} = \begin{pmatrix} R(\mathbf{x}_{r,0},\mathbf{x}_{s,0}) & R(\mathbf{x}_{r,0},\mathbf{x}_{s,1}) & \ldots & R(\mathbf{x}_{r,0},\mathbf{x}_{s,N}) \\ R(\mathbf{x}_{r,1},\mathbf{x}_{s,0}) & R(\mathbf{x}_{r,1},\mathbf{x}_{s,1}) & \ldots & R(\mathbf{x}_{r,1},\mathbf{x}_{s,N}) \\ \vdots & \vdots & \ddots & \vdots \\ R(\mathbf{x}_{r,M},\mathbf{x}_{s,0}) & R(\mathbf{x}_{r,M},\mathbf{x}_{s,1}) & \ldots & R(\mathbf{x}_{r,M},\mathbf{x}_{s,N}) \end{pmatrix}$$

A column of matrix $\mathbf{R}$ contains the discretized version of $R_0(\mathbf{x},\mathbf{x}_A)$ for a fixed source position at $\mathbf{x}_s$ and a range of receiver positions $\mathbf{x}_r$ at $x_{3,0}$ (Berkhout (1982)). For a fixed spread geometry $\mathbf{R}$ is symmetric (but not Hermitian) due to the reciprocity relation between source and receiver positions.
With these definitions equation () is written in matrix notation as:

$$\mathbf{T}^H\mathbf{T} = \mathbf{I} - \mathbf{R}^H\mathbf{R}, \quad (5)$$

where superscript ^{H} denotes complex-conjugate transpose and $\mathbf{I}$ is an identity matrix. Note that correlation matrices are Hermitian:

$$(\mathbf{T}^H\mathbf{T})^H = \mathbf{T}^H\mathbf{T}$$

The representation of the transmission response given by equation (3) is written in matrix notation as:

$$\mathbf{T} = \mathbf{W}_p\mathbf{C}, \quad (6)$$

Substituting equation (6) into equation (5) and making use of the fact that for the primary propagator we can write $\mathbf{W}_p^H\mathbf{W}_p \approx \mathbf{I}$, gives

$$\mathbf{C}^H\mathbf{C} = \mathbf{I} - \mathbf{R}^H\mathbf{R}. \quad (7)$$

This equation states that the auto-correlation of the transmission coda matrix is related to the auto-correlation of the reflection matrix.

Eigenvalues Λ

To solve $\mathbf{C}$ from $\mathbf{C}^H\mathbf{C}$ the assumption is made that it is possible to decompose $\mathbf{C}$ in its eigenvalues;

$$\mathbf{C} = \mathbf{L}\mathbf{\Lambda}_c\mathbf{L}^H \quad (8)$$

with

$$\mathbf{\Lambda}_c = \exp\{\mathbf{A}\Delta x_3\} = \begin{pmatrix} e^{-\mathcal{A}_1\Delta x_3} & 0 & \ldots & 0 \\ 0 & e^{-\mathcal{A}_2\Delta x_3} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & e^{-\mathcal{A}_N\Delta x_3} \end{pmatrix} \quad (9)$$

and $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_N$ being the temporal Fourier transforms of causal filters. Using the assumption of equation (8) the correlation of the coda matrix of equation (7) can be written as:

$$\mathbf{C}^H\mathbf{C} = \mathbf{L}\mathbf{\Lambda}_c^H\mathbf{\Lambda}_c\mathbf{L}^H \quad (10)$$

$$\mathbf{\Lambda}_c^H\mathbf{\Lambda}_c = \exp\{-2\Re\{\mathbf{A}\}\Delta x_3\} \quad (11)$$

$$\mathbf{\Lambda}_c^H\mathbf{\Lambda}_c = \begin{pmatrix} e^{-2\Re\{\mathcal{A}_1\}\Delta x_3} & 0 & \ldots & 0 \\ 0 & e^{-2\Re\{\mathcal{A}_2\}\Delta x_3} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & e^{-2\Re\{\mathcal{A}_N\}\Delta x_3} \end{pmatrix}. \quad (12)$$

Note, the coda can be written as $C(p,\Delta x_3) = \exp(-\mathcal{A}(p)\Delta x_3)$ With these assumptions the relation between $\mathbf{C}^H\mathbf{C}$ and $\mathbf{R}^H\mathbf{R}$ can be written more explicitely

$$\begin{aligned} \mathbf{C}^H\mathbf{C} &= \mathbf{I} - \mathbf{R}^H\mathbf{R} \\ \mathbf{C}^H\mathbf{C} &= \mathbf{I} - \mathbf{L}\mathbf{\Lambda}_r\mathbf{L}^H \\ \mathbf{L}\mathbf{\Lambda}_c^H\mathbf{\Lambda}_c\mathbf{L}^H &= \mathbf{L}[\mathbf{I} - \mathbf{\Lambda}_r]\mathbf{L}^H \\ \exp\{-2\Re\{\underset{\sim}{\mathbf{A}}\}\Delta x_3\} &= \mathbf{I} - \mathbf{\Lambda}_r \\ \Re[\underset{\sim}{\mathbf{A}}] &= -\frac{1}{2\Delta x_3}\ln\{\mathbf{I} - \mathbf{\Lambda}_r\} \end{aligned}$$

For plane waves in 1D media $\mathbf{C}$ is a circulant matrix, which eigenvalues can be found by using the Fourier transform: $\mathbf{C} = \mathbf{F}^H\mathbf{\Lambda}_c\mathbf{F}$. The elements $\mathcal{A}_l$ of $\mathbf{A}$ are the Fourier transforms of causal filters in the time domain. This statement is true for 1D media, since in that case $\mathbf{A}$ is the spatial Fourier transform of $\mathbf{C}$, so $A_l = \tilde{\mathcal{A}}(k_{x,l},\omega)$. Note that before the temporal Fourier transforms yields the causal filters, the diagonal elements must be scaled from wavenumber k_x, which corresponds to a certain eigenvalue number l, to ray parameter p with $\frac{1}{\omega}(k_x = p\omega)$.
Reconstruction of $\mathcal{A}_i$ from its real part $\Re\{\mathcal{A}_i\}$ is possible because $\mathcal{A}_i$ is a causal function and with the definition of the analytic signal (Hilbert transform) it can be reconstructed (see Bracewell (1986) for more details).

$$A(\omega) = \Re\{A(\omega)\} + \frac{1}{j\pi}\int_{-\infty}^{+\infty}\frac{\Re\{A(\omega')\}}{\omega - \omega'}d\omega'$$

In practice $\mathcal{A}(p,\omega)$ is transformed back to time with a complex to complex Fourier transform $a_r(p,t) = \mathcal{F}_{\omega\to t}\{\mathcal{A}(p,\omega)\}$ and multiplied with the Heaviside function to get the causal signal.
The coda can now be calculated with: $C(p,\omega,\Delta z) = \exp(-\mathcal{A}(p,\omega)\Delta z)$.

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To summarize the procedure, the following steps must be taken to compute the transmission coda from reflection data:

$$\mathbf{R} \rightarrow \boxed{\mathbf{R}^H\mathbf{R}} \rightarrow \boxed{\mathbf{L}\Lambda_r\mathbf{L}^H} \rightarrow \boxed{\Lambda_c^H\Lambda_c} \rightarrow \boxed{\mathbf{A}} \rightarrow \boxed{\exp\{-\mathbf{A}\Delta x_3\}} \rightarrow \mathbf{C}$$

where Λ_r contains the eigenvalues of $\mathbf{R}^H\mathbf{R}$ and $\mathbf{I} - \Lambda_r = \Lambda_c^H\Lambda_c$.
The inverse of the transmission coda (see also Herman (1992), Wapenaar and Hermann (1993)), in combination with an inverse primary propagator, may be used in seismic reflection imaging to obtain an image in which the internal multiple scattering effects are suppressed. In this approach the inverse primary propagator is estimated from the traveltime data (as usual), whereas the inverse transmission coda is obtained from the cross-correlation of the reflection measurements.
For comparison, in the imaging scheme proposed by Weglein et al. (2000), the full inverse operator (primaries as well as internal multiples) is estimated directly from the reflection measurements. The advantages and disadvantages of both methods with respect to accuracy, stability, etc. remain to be investigated.

Results for point and plane wave sources in 1D media

Plane waves

For 1D sources (plane waves) in 1D media $\mathbf{R}^H\mathbf{R}$ is represented by a circulant matrix. The eigenvalues of a circulant matrix can be obtained by using the Fourier transform. So the circulant matrix

$$\mathbf{C} = \begin{pmatrix} c_0 & c_{n-1} & c_{n-2} & \cdots & c_1 \\ c_1 & c_0 & c_{n-1} & \cdots & c_2 \\ c_2 & c_1 & c_0 & \cdots & c_3 \\ \vdots & \vdots & \vdots & & \vdots \\ c_{n-1} & c_{n-2} & c_{n-3} & \cdots & c_0 \end{pmatrix}$$

can be diagonalized by the DFT

$$\Lambda = \mathbf{F}\mathbf{C}\mathbf{F}^H$$

The eigenvalues of $\mathbf{C}$ are the discrete Fourier transform of the first column in $\mathbf{C}$.
To illustrate the procedure a simple flat three layer medium with velocities 1000, 4000, 1000 m/s and a layer thickness of 100 m is used. The layer thickness gives an internal multiple train (coda) of 4000/200 = 0.05 s.. The reflection response of a plane wave is shown in Figure 4. From the auto-correlation the eigenvalues are computed by using FFT's and shown in Figure 5. The same eigenvalue results could be obtained by using Lapack routines. However, it is observed that the sorting of the eigenvalues is different than from the FFT.

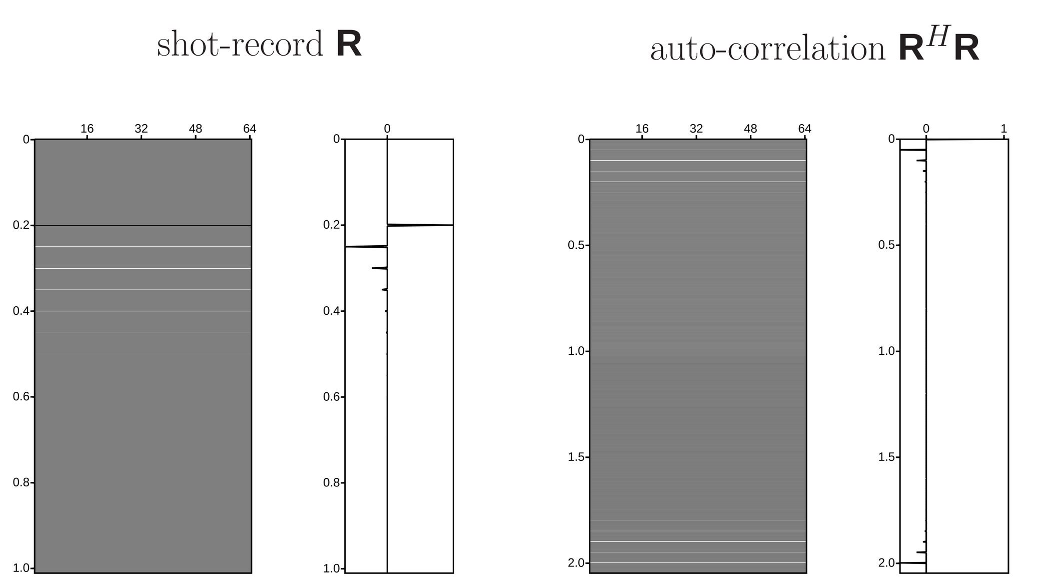

Figure 4: *Reflection response and the auto correlation of the refelction response for a plane wave in lateral invariant media. Note the train of internal multiples after the first reflection.*

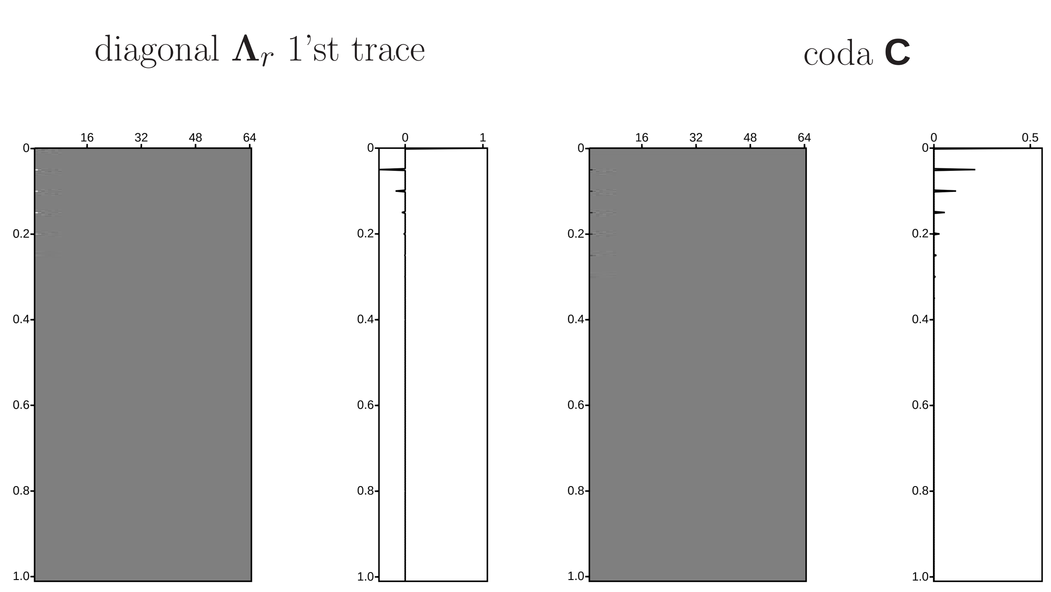

Figure 5: *Eigenvalue decomposition of the autocorrelation of the reflection response. For this simple medium there is only one eigenvalue at the first position. This eigenvalue indicates the plane wave at zero incidence. From this eigenvalue decompositon the coda can be calculated and is shown on the right.*

Point sources

For point sources in 1D media $\mathbf{R}^H\mathbf{R}$ is Hermitian and Toeplitz. Using the Toeplitz matrix as a basis to construct a circulant matrix, an approximation of its eigenvalues can be obtained by using the Fourier transform.

A $m \times n$ Toeplitz matrix

$$\mathbf{T} = \begin{pmatrix} x_m & x_{m+1} & & \cdots & x_{m+n-1} \\ x_{m-1} & x_m & x_{m+1} & & \vdots \\ \vdots & x_{m-1} & x_m & \ddots & \vdots \\ \vdots & & x_{m-1} & \ddots & x_{m+1} \\ \vdots & & & \ddots & x_m \\ \vdots & & & & x_{m-1} \\ \vdots & & & & \vdots \\ x_1 & & & & x_n \end{pmatrix}$$

can be represented by

$$T_{col} = \begin{pmatrix} x_m \\ x_{m-1} \\ \vdots \\ x_1 \end{pmatrix}, T_{row} = \begin{pmatrix} x_{m+n-1} \\ x_{m+n-2} \\ \vdots \\ x_{m+1} \end{pmatrix}$$

A circulant matrix which contains a 5×5 Toeplitz matrix has the following form:

$$C = \begin{pmatrix} T_{col} \\ 0 \\ T_{row} \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} x_3 & x_4 & x_5 & 0 & 0 & 0 & x_1 & x_2 \\ x_2 & x_3 & x_4 & x_5 & 0 & 0 & 0 & x_1 \\ x_1 & x_2 & x_3 & x_4 & x_5 & 0 & 0 & 0 \\ 0 & x_1 & x_2 & x_3 & x_4 & x_5 & 0 & 0 \\ 0 & 0 & x_1 & x_2 & x_3 & x_4 & x_5 & 0 \\ 0 & 0 & 0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ x_5 & 0 & 0 & 0 & x_1 & x_2 & x_3 & x_4 \\ x_4 & x_5 & 0 & 0 & 0 & x_1 & x_2 & x_3 \end{pmatrix} \qquad (12)$$

The similar 1D 3 layer medium is chosen with layer velocities 1000,4000 and 1000 m/s and interface depths 0,200,400 m. The middle layer, with a thickness of 200 m, will give internal multiples which are 0.1 s. apart on the zero-offset trace. The reflection and transmission data, together with their correlation, are shown in Figure 1.

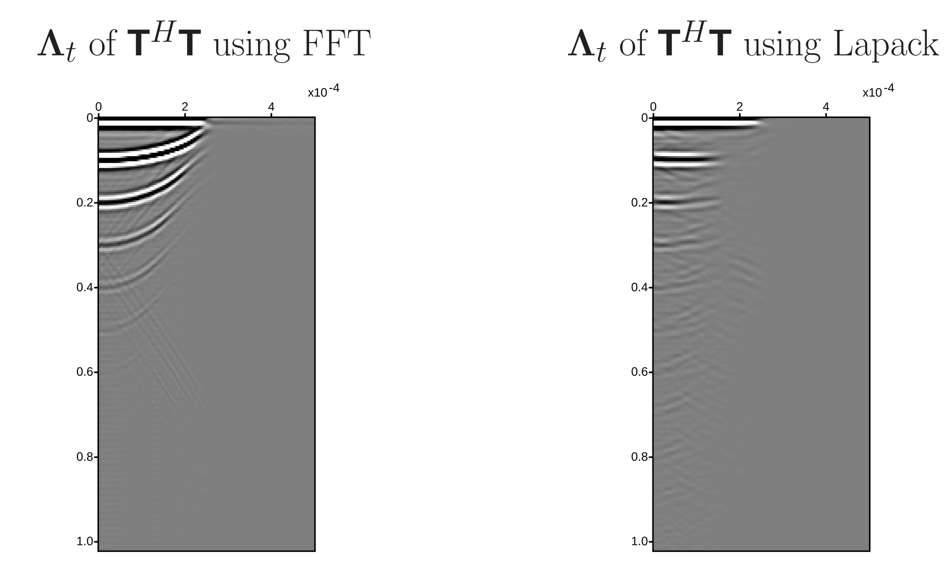

Figure 6: *Eigenvalues (Λ_t) of the correlation of transmission response in $\tau - p$ domain. In the left picture eigenvalues are calculated using FFT's on a Toeplitz matrix embedded in a ciculant matrix. The right hand side is calculated using Lapack routines on the Toeplitz matrix directly.*

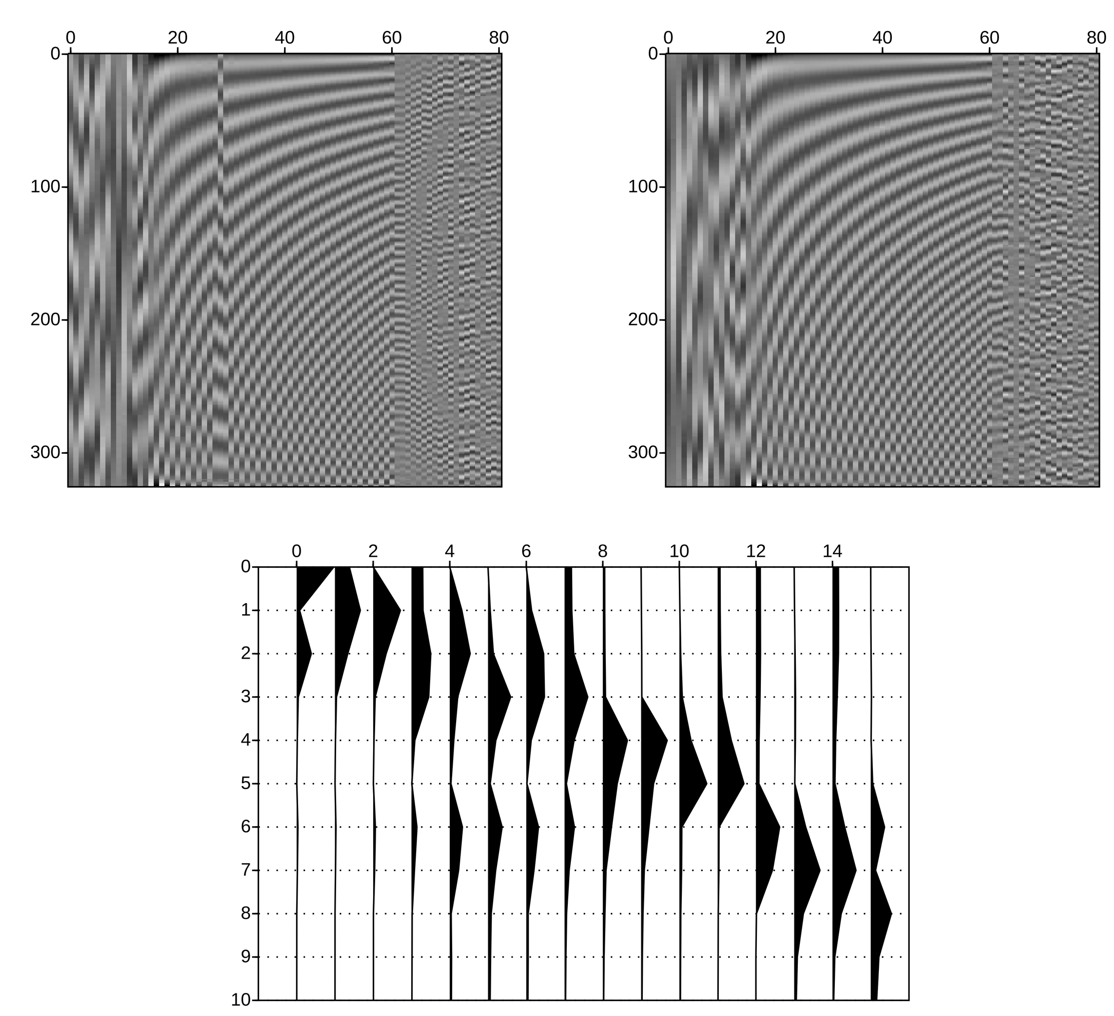

Figure 7: *The eigenvalues must be sorted before the transformation to $\tau - p$ can be done. The sorting carried out is based on the frequency content of the eigenvector. In this figure the eigenvectors are shown for a frequency of 50 Hz.*

The eigenvalues of $\mathbf{T}^H\mathbf{T} = \mathbf{I} - \mathbf{R}^H\mathbf{R}$ as function of the ray parameter p are shown in Figure 6. The eigenvalues were computed in two different ways. The procedure as explained above was used to embed the Hermitian Toeplitz into a circulant matrix and the Fourier transform to calculate the eigenvalues. In the right hand side picture Lapack routines were used to compute the eigenvalues of $\mathbf{T}^H\mathbf{T}$. Note the differences between the two pictures.
After the eigenvalue decomposition the eigenvalues must be mapped to corresponding wavenumbers to be able to transform the results to the ray-parameter domain. The mapping from eigenvalues to wavenumber requires a sorting of the eigenvalues and illustrated in Figure 7. Different sorting algorithms have been tried, but the one based on the frequency content of the eigenvectors gives the best results.
Figure 8 shows the eigenvalues in the $\tau - p$ domain for different computation methods. Figure 8a is based on the circulant matrix and FFT's. The Lapack routine zheevx is also used to calculate the eigenvalues of the circulant matrix and gives, as expected, exactly the same results as obtained with the Fourier transform (Figure 8b). The sorting used to obtain Figure 8b is based on the frequency content of the eigenvectors. In Figure 8c the eigenvalues of the Toeplitz matrix are computed with the same Lapack routine zheevx. The effects of the edges of the matrix and/or the incorrect sorting of the eigenvalues are visible in the eigenvalue result.
Figure 8d shows the computed coda operator from the eigenvalues of the circulant matrix. To compute the coda the logarithm of the eigenvalues, the diagonal of $\Lambda_c^H\Lambda_c$, is taken yielding $-2\Re\{\mathcal{A}_i\}$. From the real part of $\mathcal{A}_i$ the causal function is calculated using the Hilbert transform and the result is inserted into equation (4) to calculate the coda.

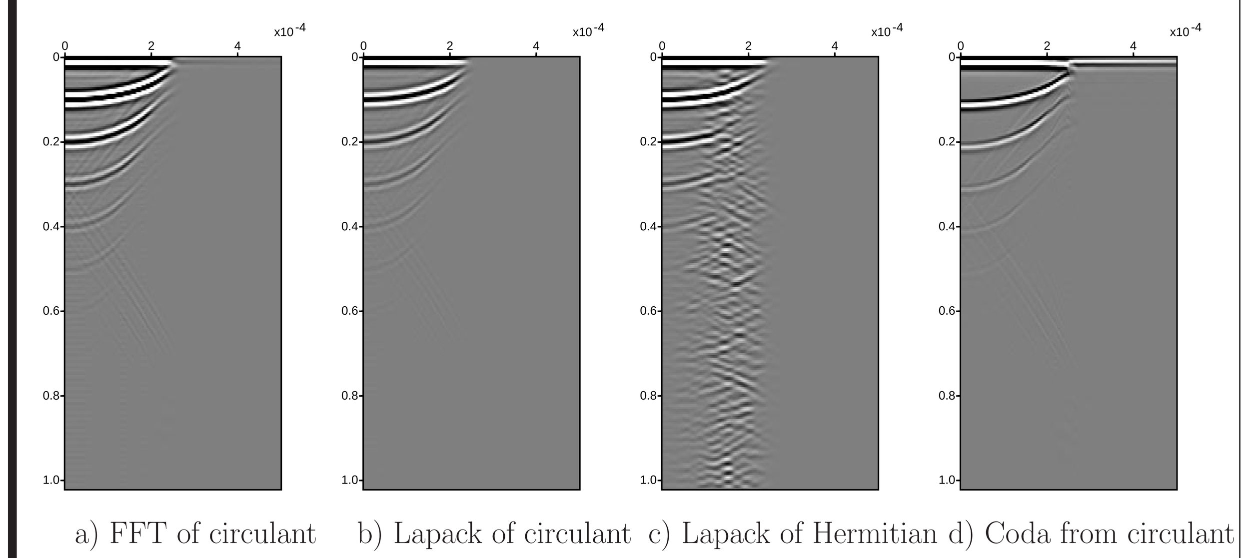

Figure 8: *Eigenvalues (Λ_t) of correlation of transmission response in $\tau - p$ domain using FFT's (a) and Lapack routines (b) for a circulant matrix based on the Toeplitz matrix $\mathbf{T}^H\mathbf{T}$. The eigenvalue decomposition of the Hermitian matrix (c) is done with Lapack. The coda operator (d) is calculated from the eigenvalue decomposition based on the circulant matrix.*

Practical problems

$$\mathbf{R}(\mathbf{x}) \Rightarrow \mathbf{R}^H\mathbf{R} \Rightarrow \mathbf{L}\Lambda_r\mathbf{L}^H \Rightarrow \Lambda_r(p) \Rightarrow \mathbf{A}(p) \Rightarrow \exp\{-\mathbf{A}(p)\} \Rightarrow \mathbf{T}(p)$$

- Computation of $\mathbf{I} - \mathbf{R}^H\mathbf{R}$
- Ordering of eigenvalues (per ω) and mapping to k_x
- Check eigenvectors: $\mathbf{L}^H\mathbf{L} = \mathbf{I}$
- Eigenvector analysis for 2D media
- Correct scaling of amplitudes
- Windowing of R; below $\partial\mathcal{D}_m$ heterogeneous media
- Construction of W_g from C and W_p

Conclusions

The implicit relation between correlated reflection and correlated transmission data can be used to estimate the transmission coda for 3D inhomogeneous media. The used assumptions on the transmission operator led to an eigenvalue problem which can be solved numerically. For plane waves and point sources in 1D media the procedure has been demonstrated. Further investigation is needed in order to understand the eigenvalue decomposition in laterally varying media, the influence of the acquisition geometry and the sensitivity to amplitude errors.

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