From Reflection to Transmission Data

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Contents and Goal

- Brief introduction Reciprocity Theorems
- From R to T for 1D media
- From R to T for 2D media
- Conclusions

Goal: Using calculated coda in reflection imaging to suppress the effects of internal multiples.

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One-way Reciprocity Theorems

Convolution type:

$$\int_{\partial\mathcal{D}_0} \{P_A^+ P_B^- - P_A^- P_B^+\} d^2\mathbf{x} = \int_{\partial\mathcal{D}_m} \{P_A^+ P_B^- - P_A^- P_B^+\} d^2\mathbf{x}$$

Correlation type:

$$\int_{\partial\mathcal{D}_0} \{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\} d^2\mathbf{x} = \int_{\partial\mathcal{D}_m} \{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\} d^2\mathbf{x}$$

See article *"Relations between reflection and transmission responses of 3-D in-homogeneous media."* by Kees Wapenaar, Jan Thorbecke, Deyan Dragonov 2004, Geoph. J. Int. Vol 156, p. 179-194

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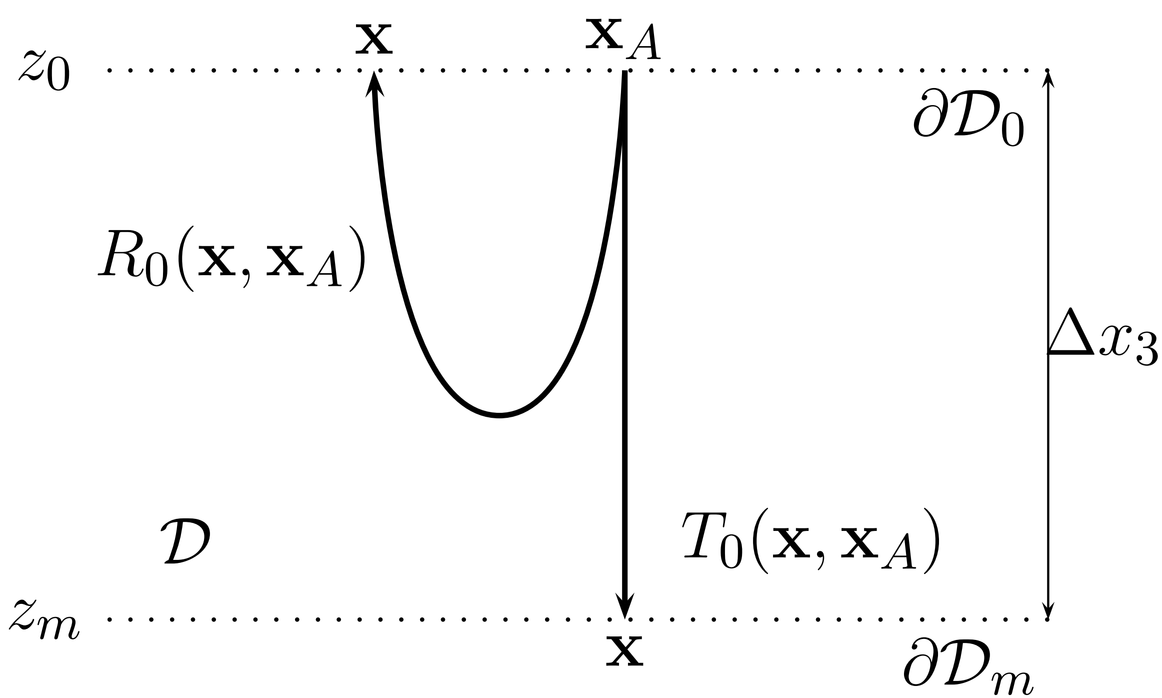
One-way Reciprocity Theorems

Correlation type:

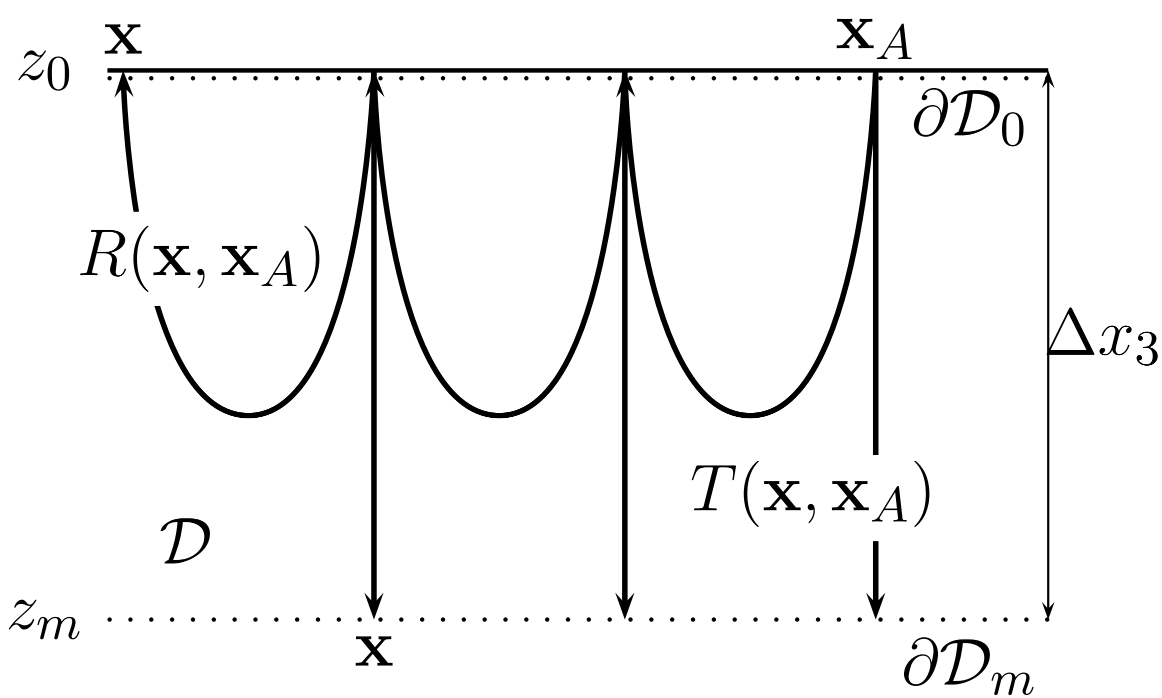
$$\int_{\partial\mathcal{D}_0}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}d^2\mathbf{x} = \int_{\partial\mathcal{D}_m}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}d^2\mathbf{x}$$



One-way Reciprocity Theorems

Correlation type:

$$\int_{\partial\mathcal{D}_0}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}d^2\mathbf{x} = \int_{\partial\mathcal{D}_m}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}d^2\mathbf{x}$$



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Passive Seismic

$$\int_{\partial\mathcal{D}_0}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}d^2\mathbf{x} = \int_{\partial\mathcal{D}_m}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}d^2\mathbf{x}$$

Surface $\partial\mathcal{D}_0$		
Field	**State A**	**State B**
P^+	$\delta(\mathbf{x}_H - \mathbf{x}_{H,A})s_A(\omega) + rP^-$	$\delta(\mathbf{x}_H - \mathbf{x}_{H,B})s_B(\omega) + rP^-$
P^-	$R(\mathbf{x}, \mathbf{x}_A, \omega)s_A(\omega)$	$R(\mathbf{x}, \mathbf{x}_B, \omega)s_B(\omega)$
Surface $\partial\mathcal{D}_m$		
P^+	$T(\mathbf{x}, \mathbf{x}_A, \omega)s_A(\omega)$	$T(\mathbf{x}, \mathbf{x}_B, \omega)s_B(\omega)$
P^-	0	0

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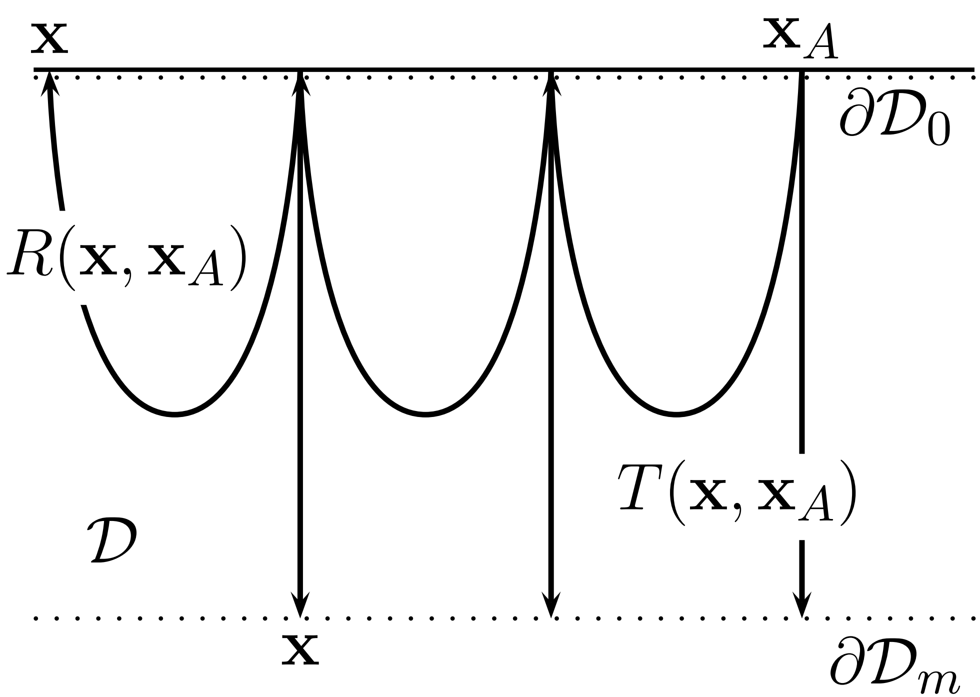
Passive Seismic

$$\delta(\mathbf{x}_{H,B} - \mathbf{x}_{H,A}) - 2\mathcal{R}[R(\mathbf{x}_A, \mathbf{x}_B)] = \int_{\partial\mathcal{D}_m} T^*(\mathbf{x}_A, \mathbf{x}) T(\mathbf{x}_B, \mathbf{x}) d^2\mathbf{x}$$



R 2 T

$$\int_{\partial\mathcal{D}_0}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}d^2\mathbf{x} = \int_{\partial\mathcal{D}_m}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}d^2\mathbf{x}$$

Surface $\partial\mathcal{D}_0$		
Field	**State A**	**State B**
P^+	$\delta(\mathbf{x}_H - \mathbf{x}_{H,A})s_A(\omega)$	$\delta(\mathbf{x}_H - \mathbf{x}_{H,B})s_B(\omega)$
P^-	$R_0(\mathbf{x},\mathbf{x}_A,\omega)s_A(\omega)$	$R_0(\mathbf{x},\mathbf{x}_B,\omega)s_B(\omega)$
Surface $\partial\mathcal{D}_m$		
P^+	$T_0(\mathbf{x},\mathbf{x}_A,\omega)s_A(\omega)$	$T_0(\mathbf{x},\mathbf{x}_B,\omega)s_B(\omega)$
P^-	0	0

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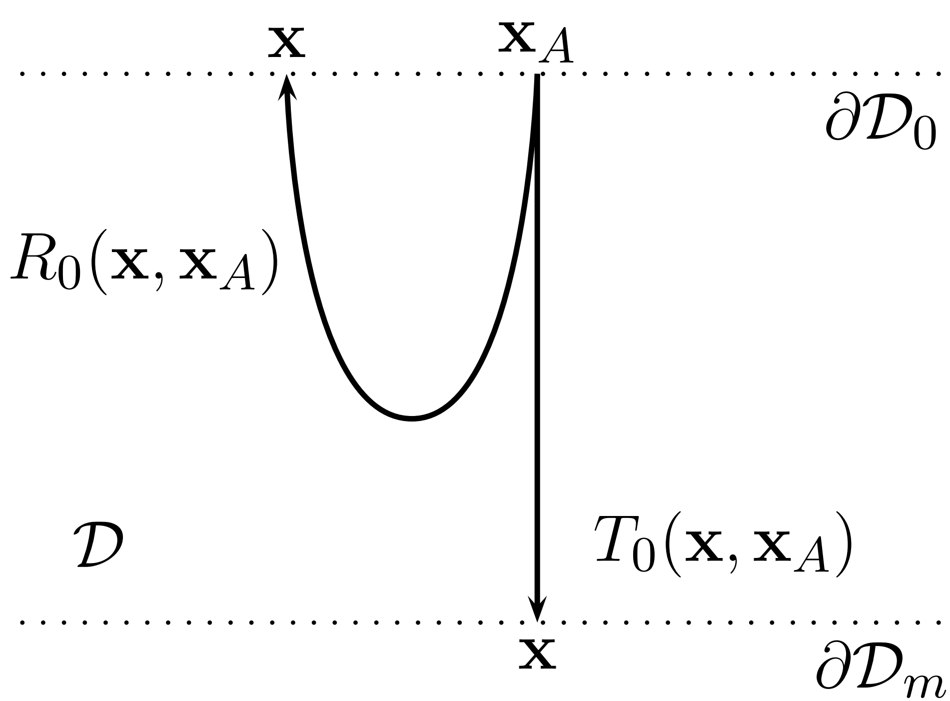
R 2 T

$$\delta(\mathbf{x}_{H,B} - \mathbf{x}_{H,A}) - \int_{\partial\mathcal{D}_0} R_0^*(\mathbf{x},\mathbf{x}_A) R_0(\mathbf{x},\mathbf{x}_B)\, d^2\mathbf{x} = \int_{\partial\mathcal{D}_m} T_0^*(\mathbf{x},\mathbf{x}_A) T_0(\mathbf{x},\mathbf{x}_B)\, d^2\mathbf{x}$$



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1D medium, 2D world

3 layer medium 1000-4000-1000 m/s thickness 200 m:
$4000/400 = 0.1$ s. internal multiple train.

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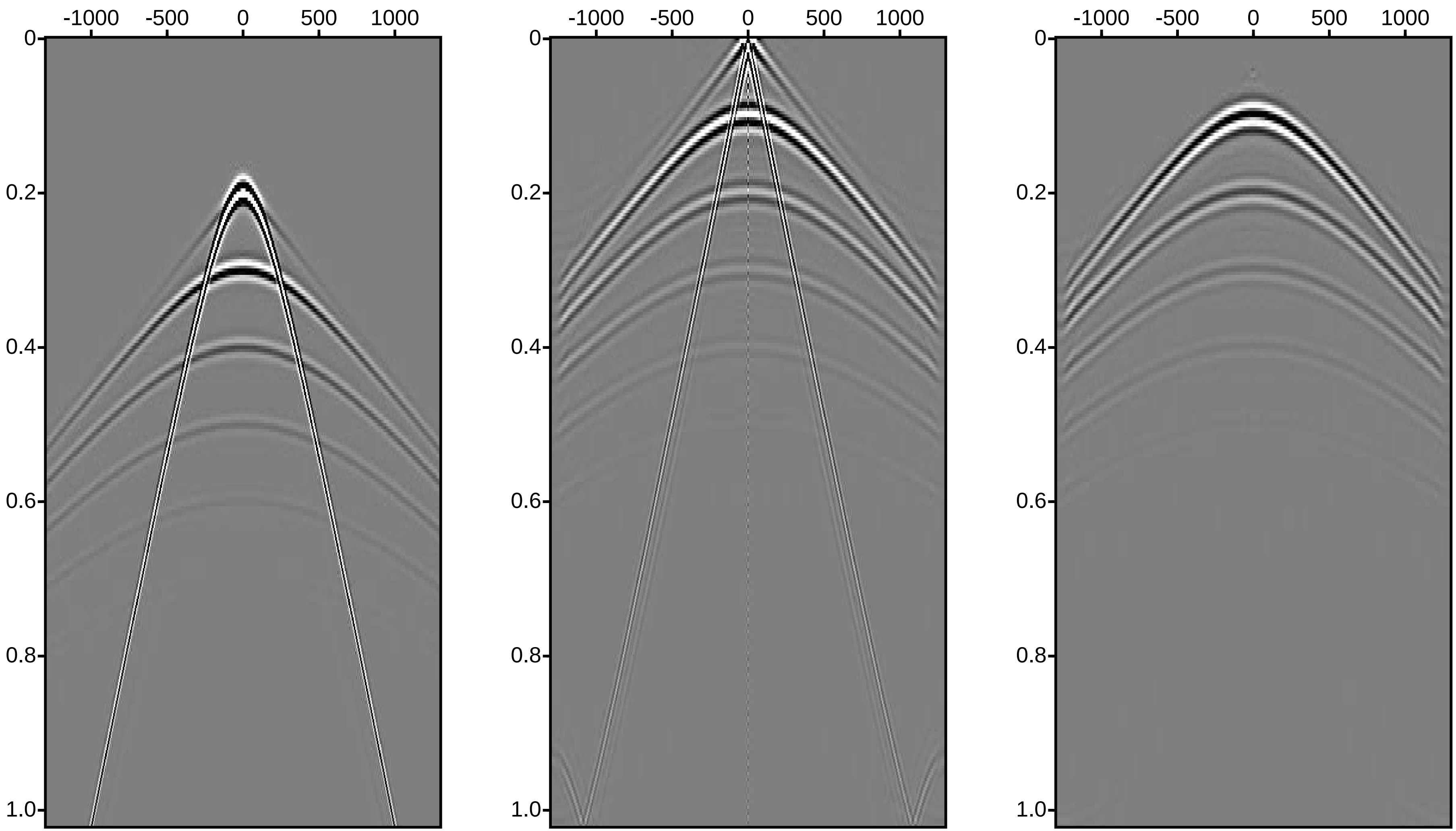
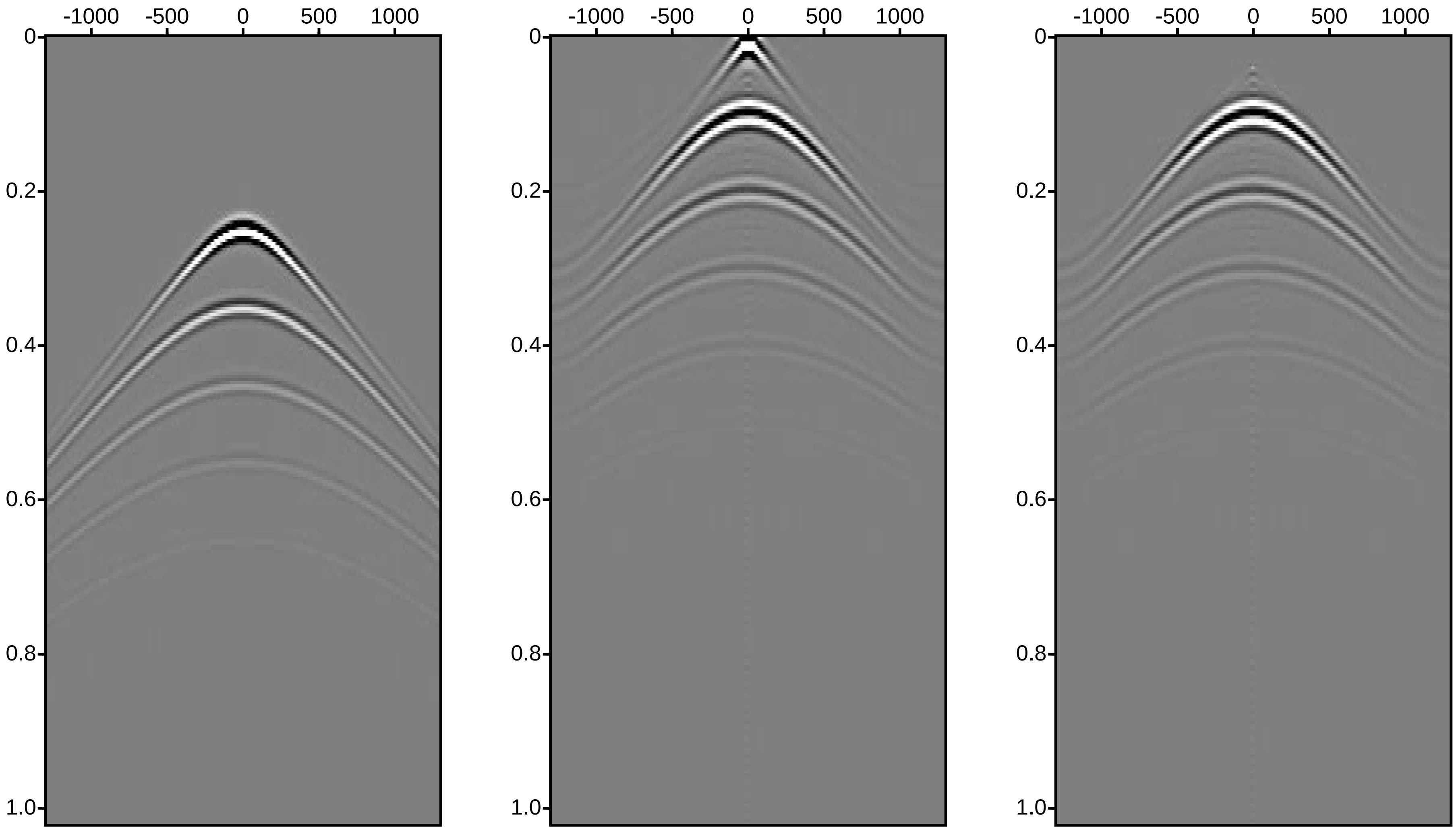
Comparison



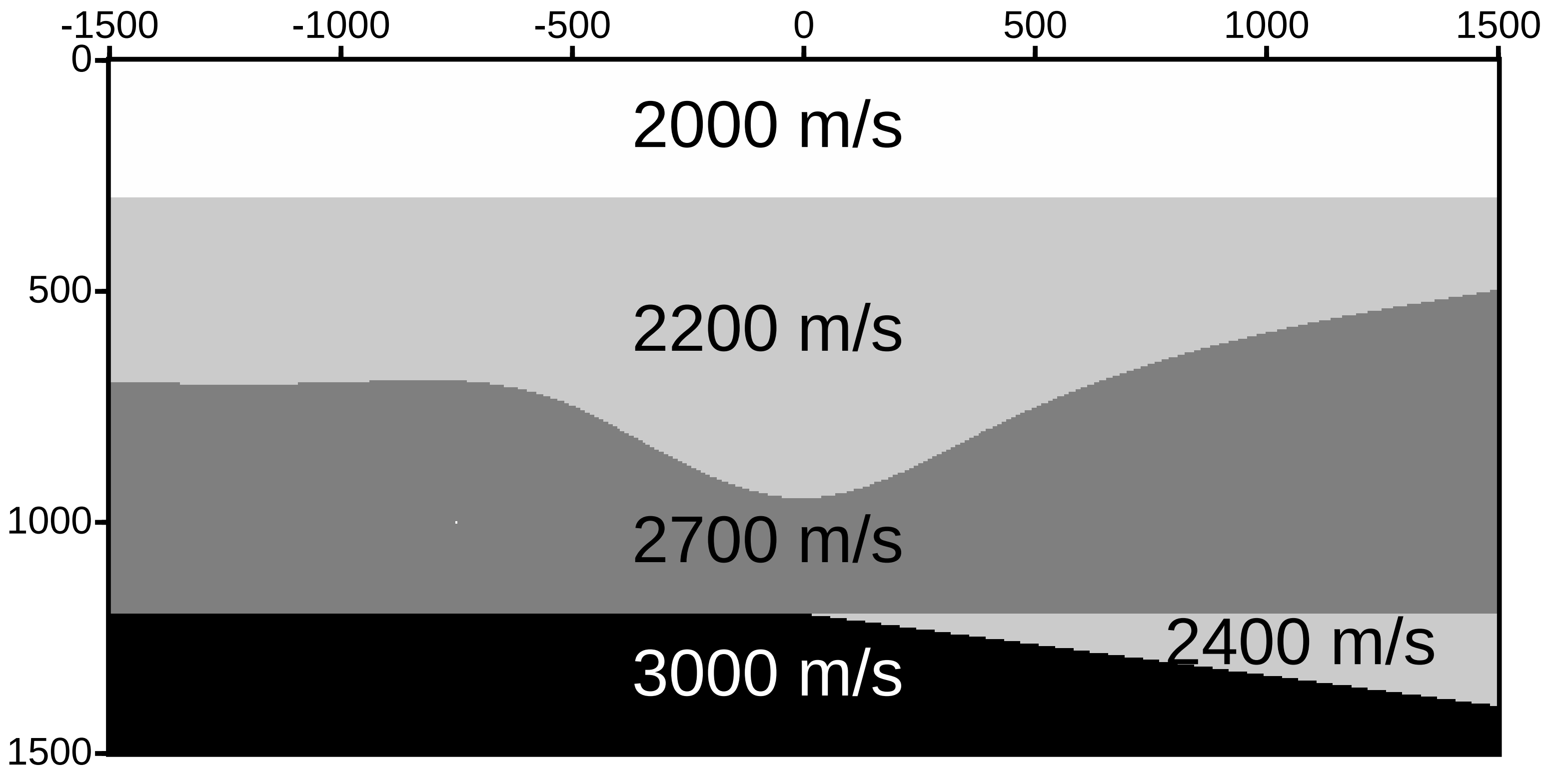
$\boldsymbol{R}_0$ $\boldsymbol{R}_0^H \mathbf{R}_0$ $\mathbf{I} - \boldsymbol{R}_0^H \mathbf{R}_0$ muted

Comparison



$\boldsymbol{T}_0$ | $\boldsymbol{T}_0^H \mathbf{T}_0$ | $\boldsymbol{T}_0^H \mathbf{T}_0$ muted

Syncline model



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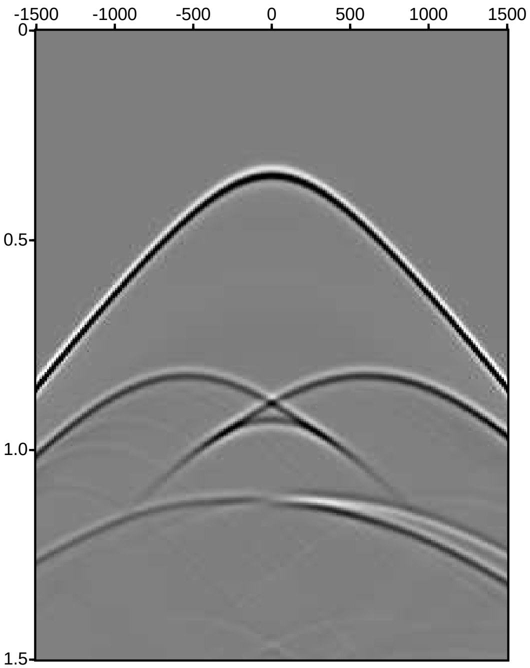
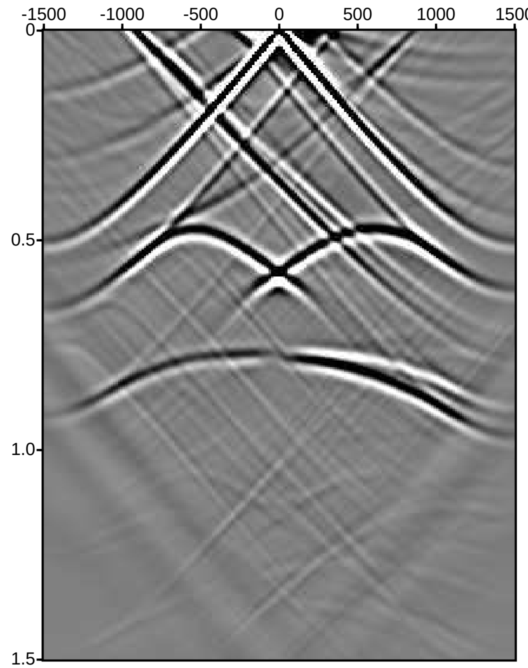
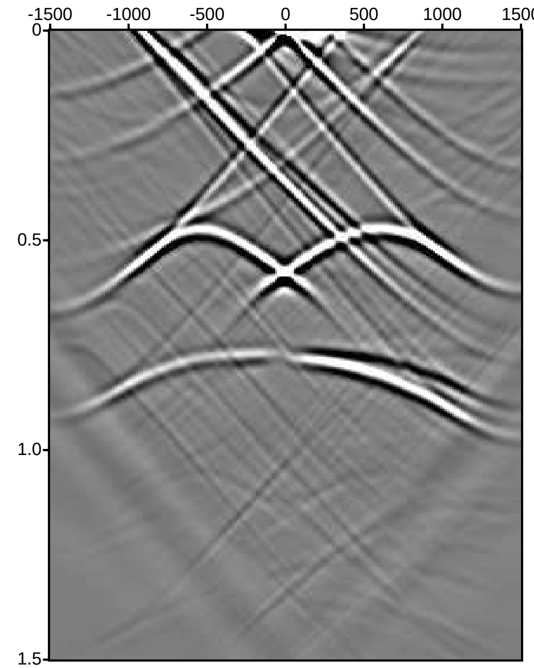
Comparison



$\boldsymbol{R}_0$



$\boldsymbol{R}_0^H \mathbf{R}_0$



$\mathbf{I} - \boldsymbol{R}_0^H \mathbf{R}_0$

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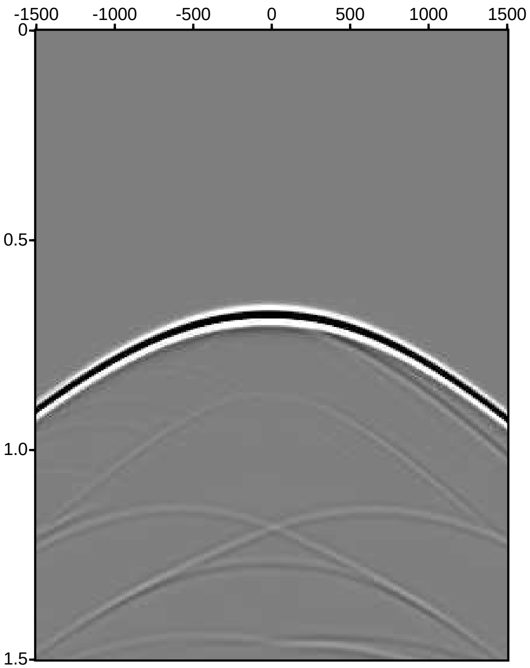
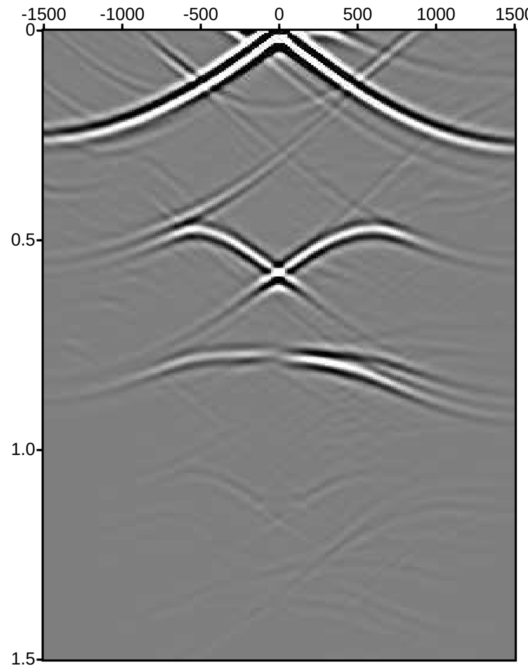
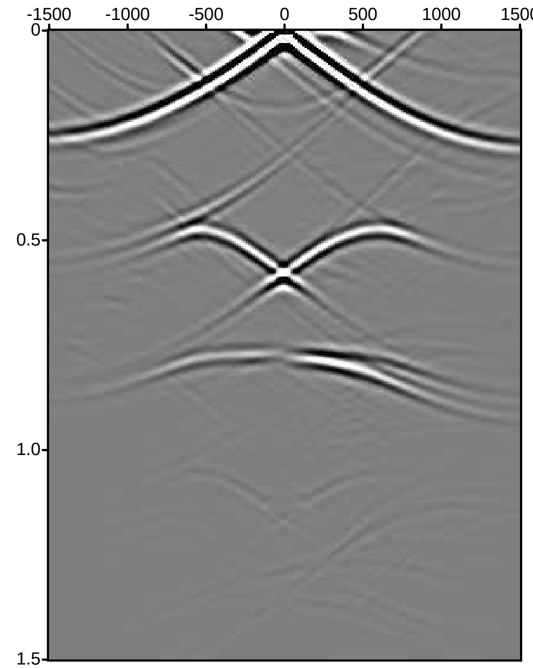
Comparison



$\boldsymbol{T}_0$



$\boldsymbol{T}_0^H \mathbf{T}_0$



$\boldsymbol{T}_0^H \mathbf{T}_0$

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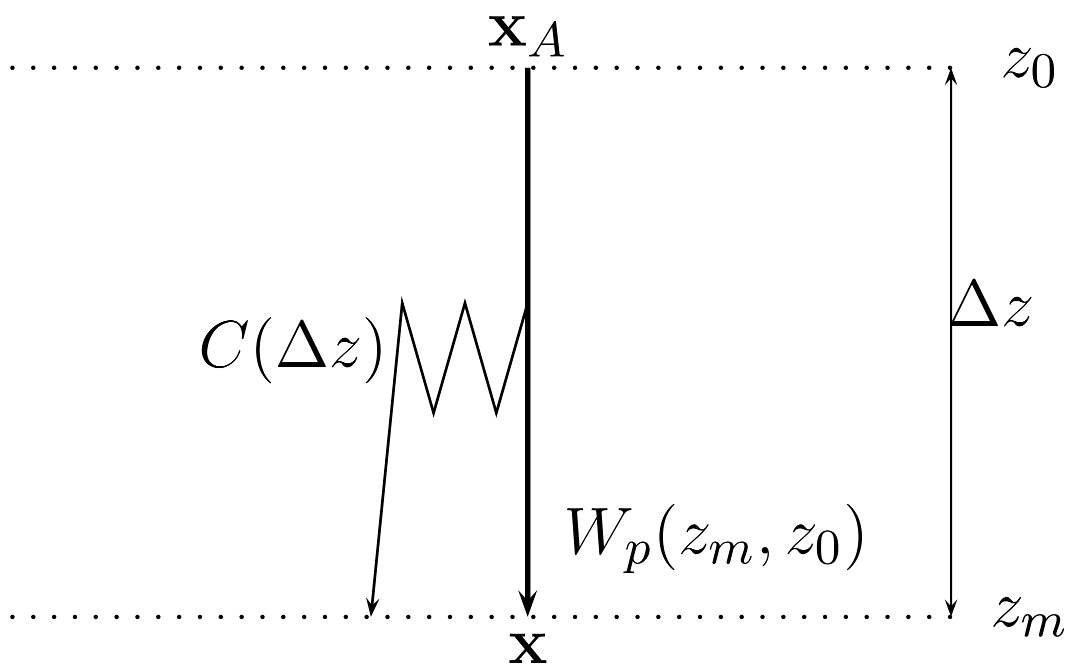
Model



$$\mathbf{T}_0(z_m, z_0) = \mathbf{W}_p(z_m, z_0)\mathbf{C}(\Delta z)$$

$$\mathbf{T}_0^H \mathbf{T}_0 = (\mathbf{W_p C})^H \mathbf{W_p C} = \mathbf{C}^H \mathbf{C} = \mathbf{I} - \mathbf{R}_0^H \mathbf{R}_0$$

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Assumptions (O'Doherty and Anstey)

$$C(p, \mathbf{x}_A, \Delta z) = \exp(-\mathcal{A}(p)\Delta z)$$

$$\mathbf{C} = \mathbf{L}\mathbf{\Lambda}_c\mathbf{L}^H$$

where

$$\mathbf{\Lambda}_c = \exp\{-\mathbf{A}\} = \begin{pmatrix} e^{-\mathcal{A}(\omega,p_1,\Delta z)} & 0 & \dots & 0 \\ 0 & e^{-\mathcal{A}(\omega,p_2,\Delta z)} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & e^{-\mathcal{A}(\omega,p_N,\Delta z)} \end{pmatrix}$$

Detour: Matrix structures

For plane waves in 1D media **C** is a circulant matrix which has the property that its Fourier transform is equal to its eigenvalues:

$$\mathbf{\Lambda}_c = \mathcal{F}_{x \to k_x}\{\mathbf{C}\}$$

$$\mathbf{C} = \mathbf{F}^H \mathbf{\Lambda}_c \mathbf{F}$$

For non-plane waves and/or 2D media the eigenvalues are computed using numerical routines from LAPACK (zgeev, zheevx).

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Eigenvalues of Matrix

Circulant (or Toeplitz) use FFT to calculate the eigenvalues:

$$\mathbf{C} = \begin{pmatrix} c_0 & c_{n-1} & c_{n-2} & \cdots & c_1 \\ c_1 & c_0 & c_{n-1} & \cdots & c_2 \\ c_2 & c_1 & c_0 & \cdots & c_3 \\ \vdots & \vdots & \vdots & & \vdots \\ c_{n-1} & c_{n-2} & c_{n-3} & \cdots & c_0 \end{pmatrix}$$

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Eigenvalues of Matrix

An $m \times n$ Toeplitz matrix can be embedded in a circulant matrix of order $m + n$ or smaller.

$$\mathbf{T} = \begin{pmatrix} x_m & x_{m+1} & & \cdots & x_{m+n-1} \\ x_{m-1} & x_m & x_{m+1} & & \vdots \\ \vdots & x_{m-1} & x_m & \ddots & \vdots \\ \vdots & & x_{m-1} & \ddots & x_{m+1} \\ \vdots & & & \ddots & x_m \\ \vdots & & & & x_{m-1} \\ \vdots & & & & \vdots \\ x_1 & & & & x_n \end{pmatrix}$$

Eigenvalues of Matrix (end detour)

Example 3×3 Toeplitz

$$\mathbf{T} = \begin{pmatrix} x_3 & x_4 & x_5 \\ x_2 & x_3 & x_4 \\ x_1 & x_2 & x_3 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} x_3 & x_4 & x_5 & 0 & 0 & 0 & x_1 & x_2 \\ x_2 & x_3 & x_4 & x_5 & 0 & 0 & 0 & x_1 \\ x_1 & x_2 & x_3 & x_4 & x_5 & 0 & 0 & 0 \\ 0 & x_1 & x_2 & x_3 & x_4 & x_5 & 0 & 0 \\ 0 & 0 & x_1 & x_2 & x_3 & x_4 & x_5 & 0 \\ 0 & 0 & 0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ x_5 & 0 & 0 & 0 & x_1 & x_2 & x_3 & x_4 \\ x_4 & x_5 & 0 & 0 & 0 & x_1 & x_2 & x_3 \end{pmatrix}$$

Computational scheme

$$\mathbf{C}^H\mathbf{C} = \mathbf{I} - \mathbf{R}_0^H\mathbf{R}_0$$

$$\mathbf{C}^H\mathbf{C} = \mathbf{I} - \mathbf{L}\Lambda_r\mathbf{L}^H$$

$$\mathbf{L}\Lambda_c^H\Lambda_c\mathbf{L}^H = \mathbf{L}[\mathbf{I} - \Lambda_r]\mathbf{L}^H$$

The eigenvalues of the cross correlation matrix have now to be mapped from wavenumber (eigenvalue number) to ray-parameter p. Then the following relation gives the real part of the causal filters:

$$\Lambda_c^H\Lambda_c = \exp\{-2\mathcal{R}\{\mathbf{A}\}\}$$

$$\exp\{-2\mathcal{R}\{\mathbf{A}\}\} = \mathbf{I} - \Lambda_r$$

$$\mathcal{R}\{\mathbf{A}\} = -\frac{1}{2}\ln\{\mathbf{I} - \Lambda_r\}$$

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Computational scheme

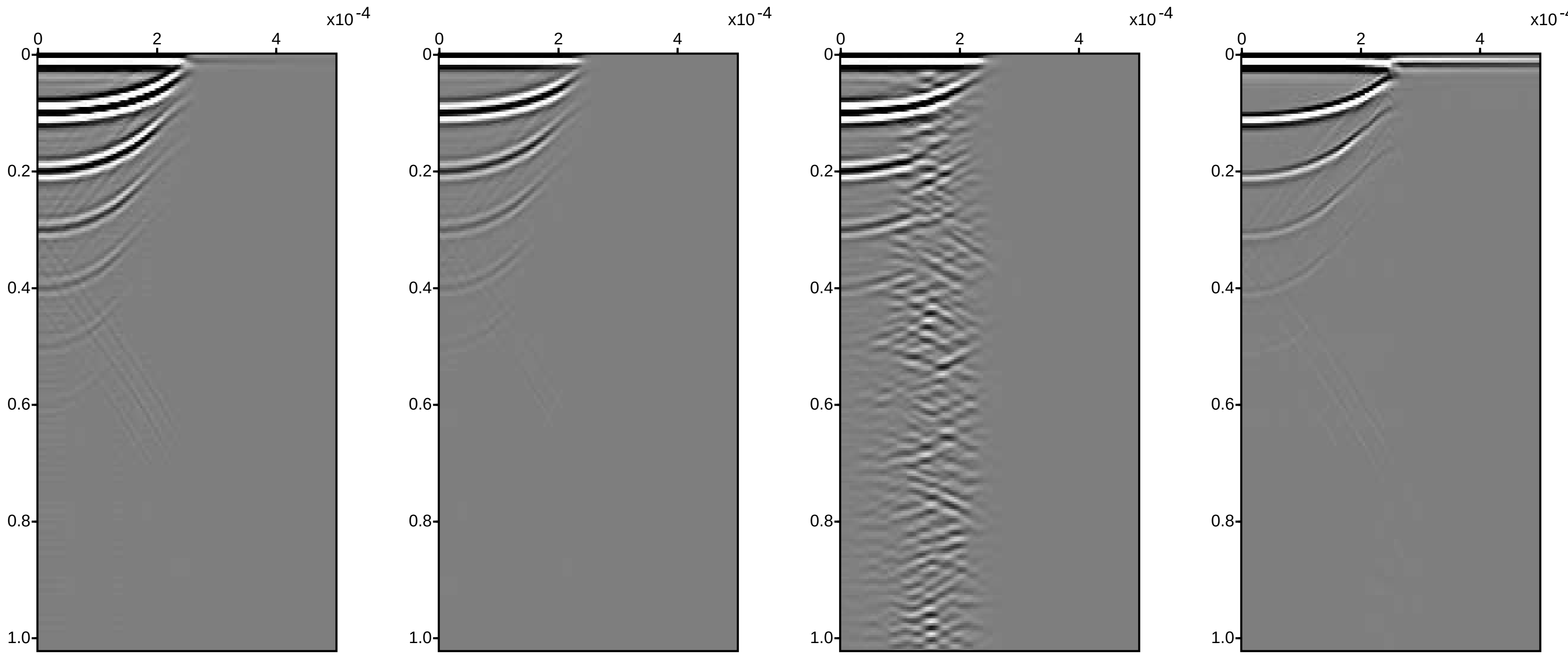
Using the Hilbert transform, the causal functions can be reconstructed from their real part, this gives $\mathcal{A}(p)$. Inserting these computed functions into equation

$$C(p, \mathbf{x}_A, \Delta z) = \exp(-\mathcal{A}(p)\Delta z).$$

Together with an estimation of the primary propagator the calculated coda can be used to calculate the transmission response $\mathbf{T}_0$ with

$$\mathbf{T}_0(z_m, z_0) = \mathbf{W}_p(z_m, z_0)\mathbf{C}.$$

Calculated Eigenvalues in 1D media



FFT | Lapack Circ. | Lapack Herm. | Coda

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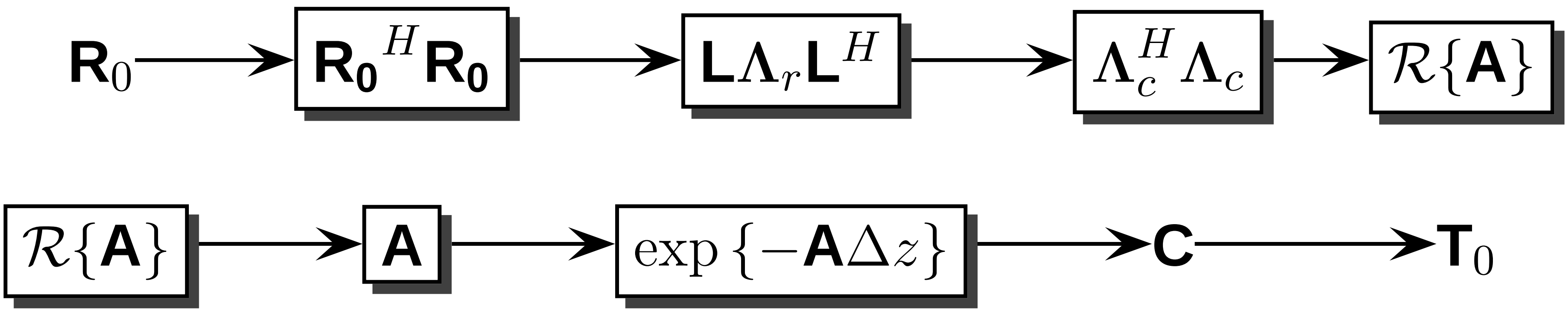
Scheme summary

To summarize the procedure, the following steps must be taken to compute the transmission coda from reflection data:



where $\mathbf{\Lambda}_r$ contains the eigenvalues of $\mathbf{R}_0^H\mathbf{R}_0$ and $\mathbf{I}-\mathbf{\Lambda}_r=\mathbf{\Lambda}_c^H\mathbf{\Lambda}_c$.

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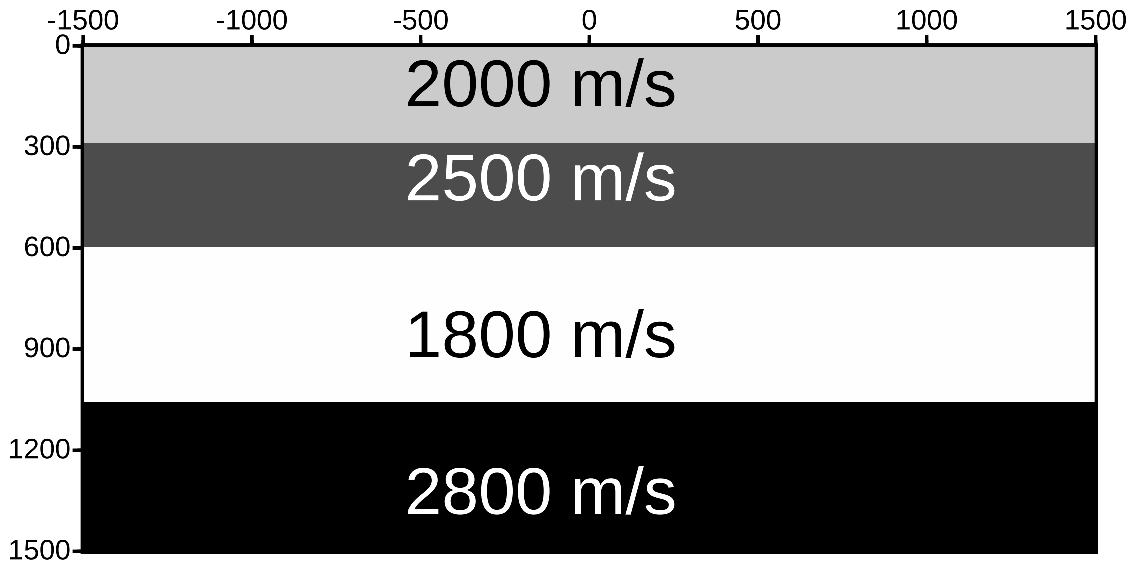
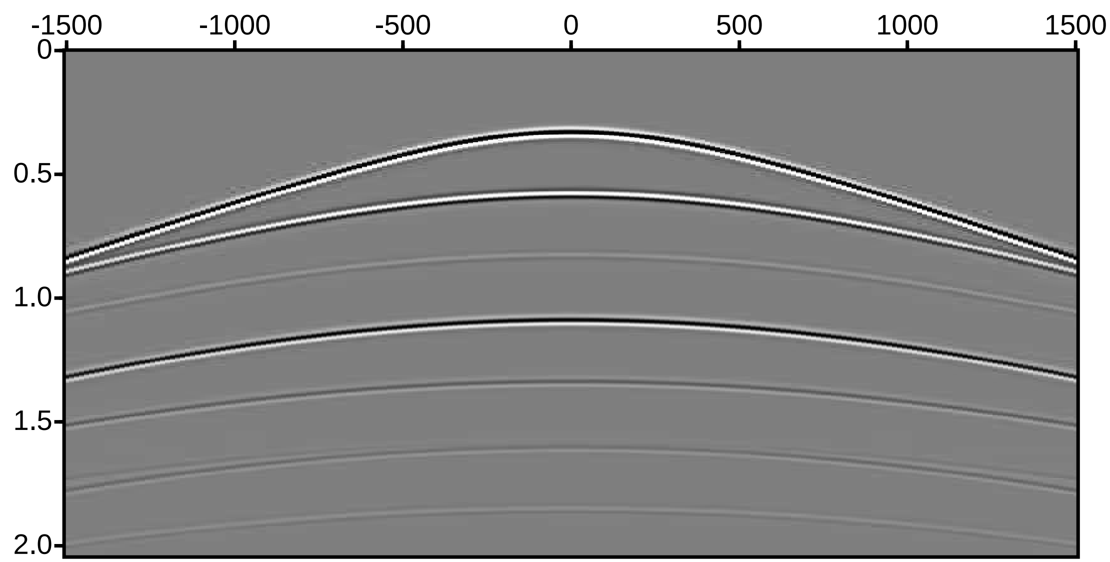
Multi layer 1D model





shot record at x=0

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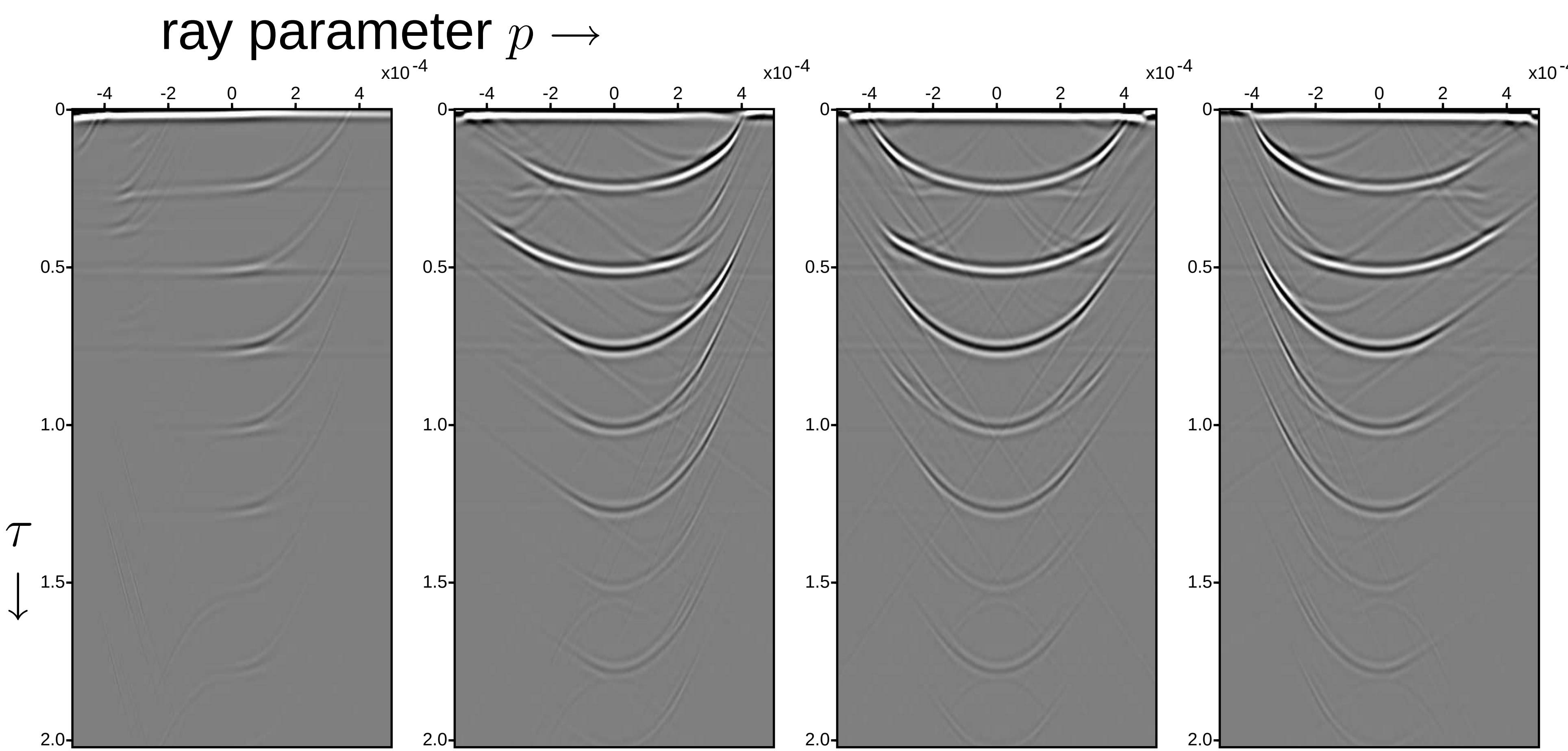
Multi layer 1D model



x=-1500 x=-750 x=0 x=750

Calculated Eigenvalues using local 1D assumption

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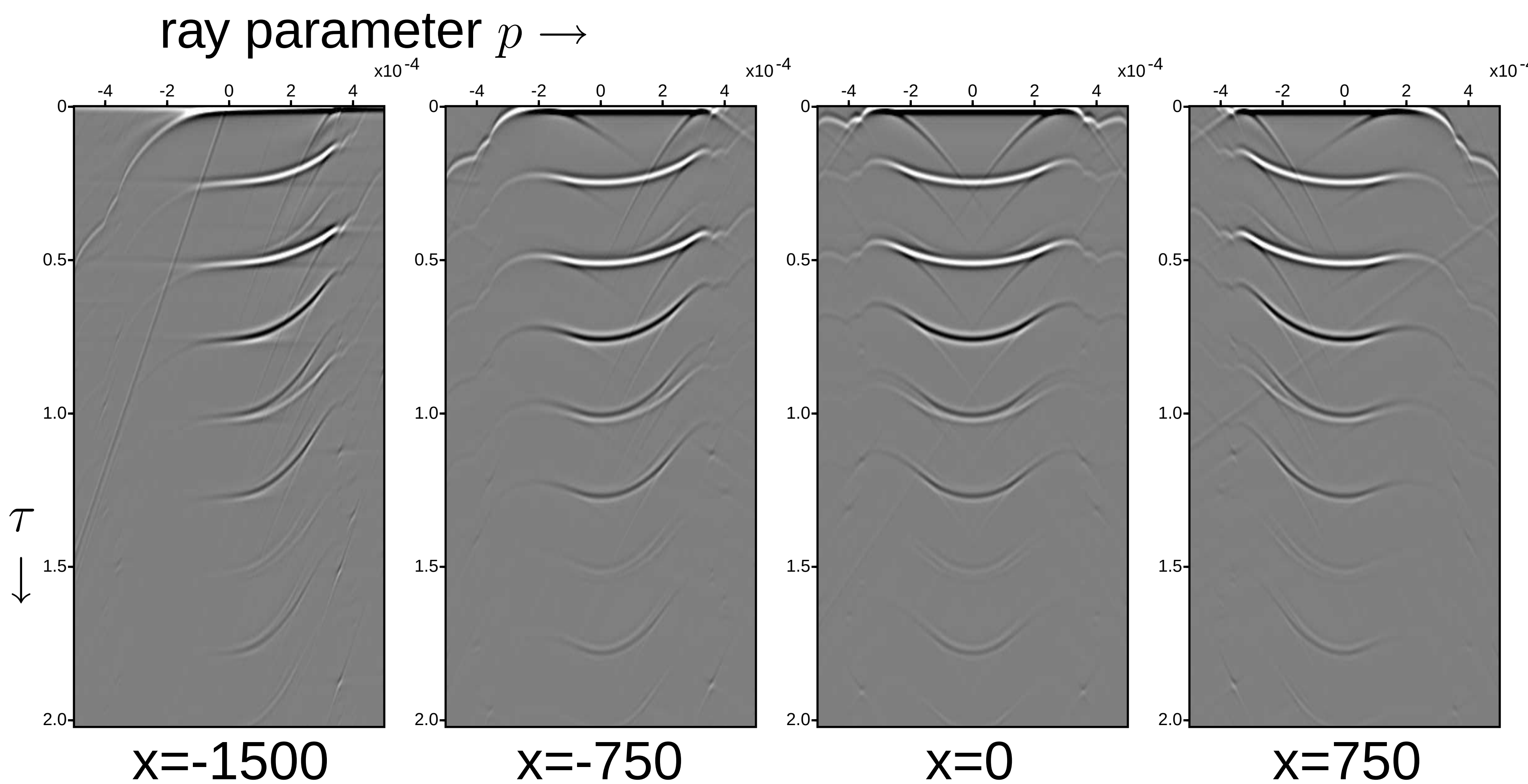
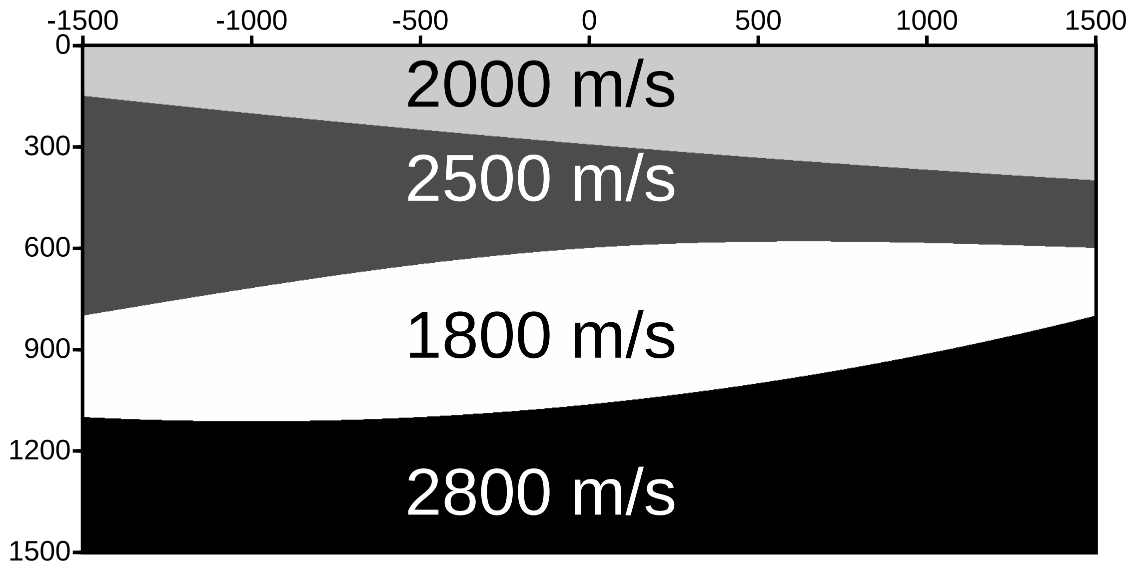
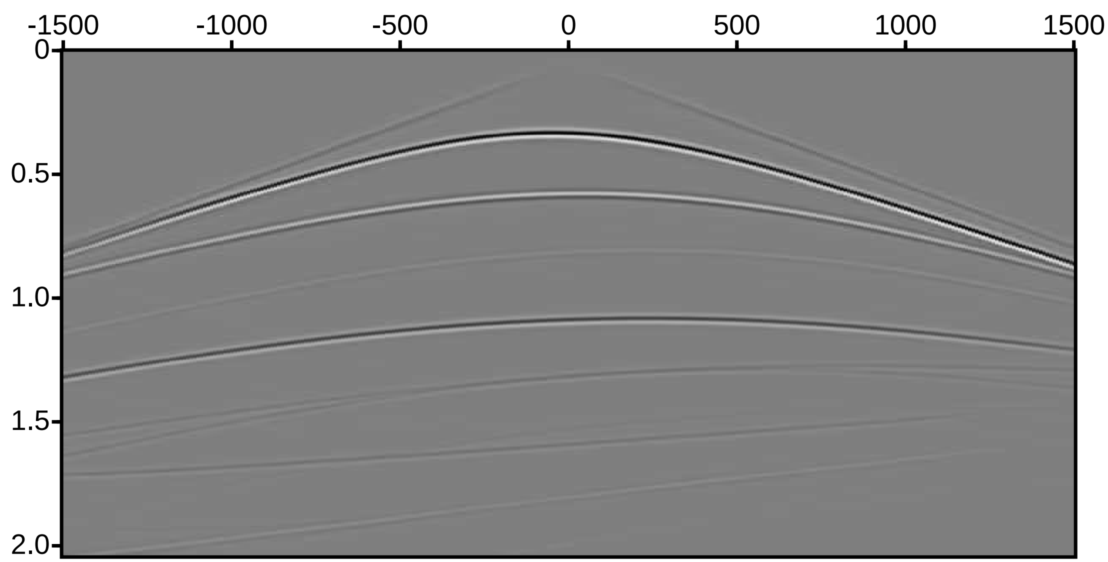
Multi layer 1D model



Modelled Transmission response

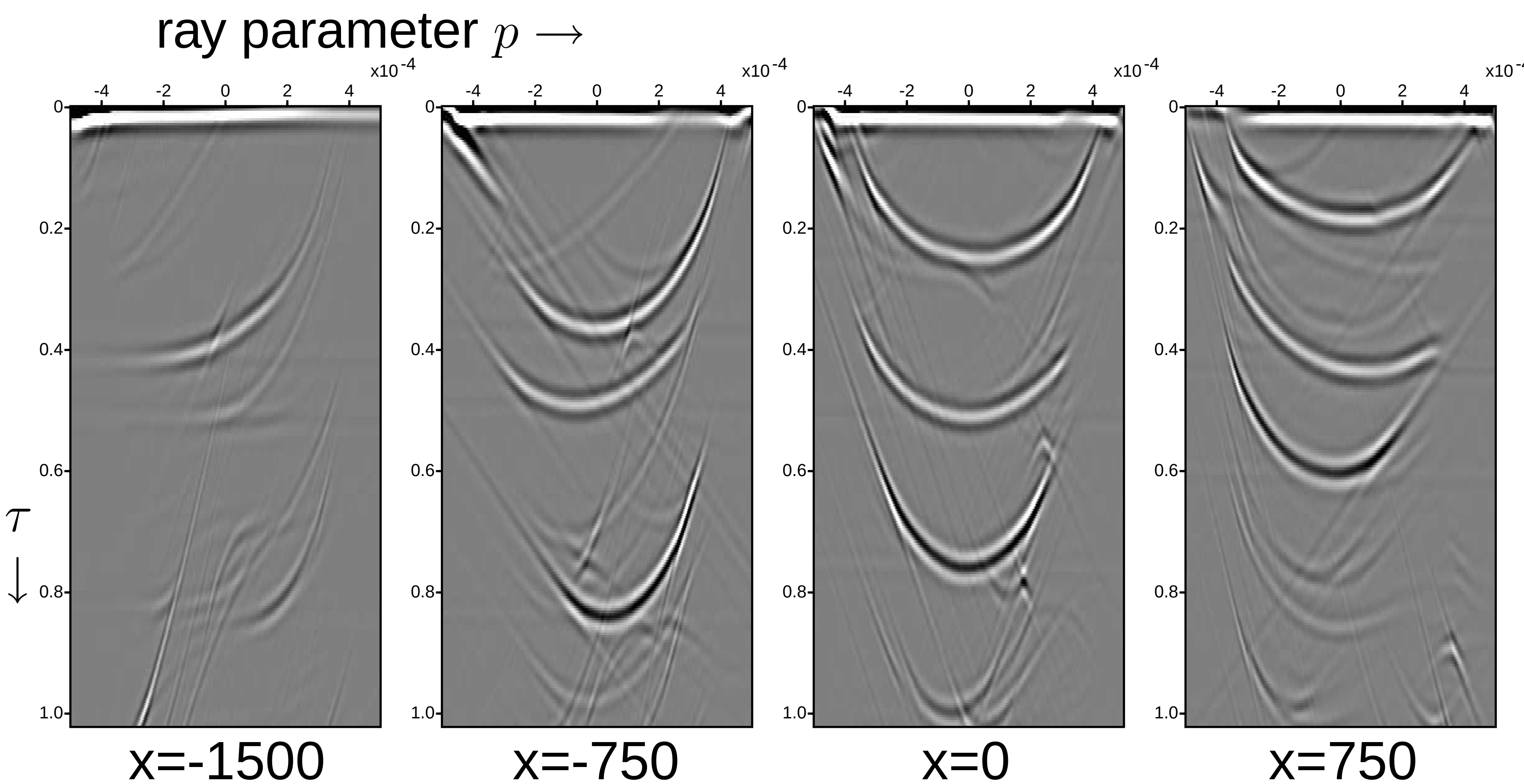
Multi layer simple 2D





shot record at x=0

Multi layer simple 2D



Calculated Eigenvalues using local 1D assumption

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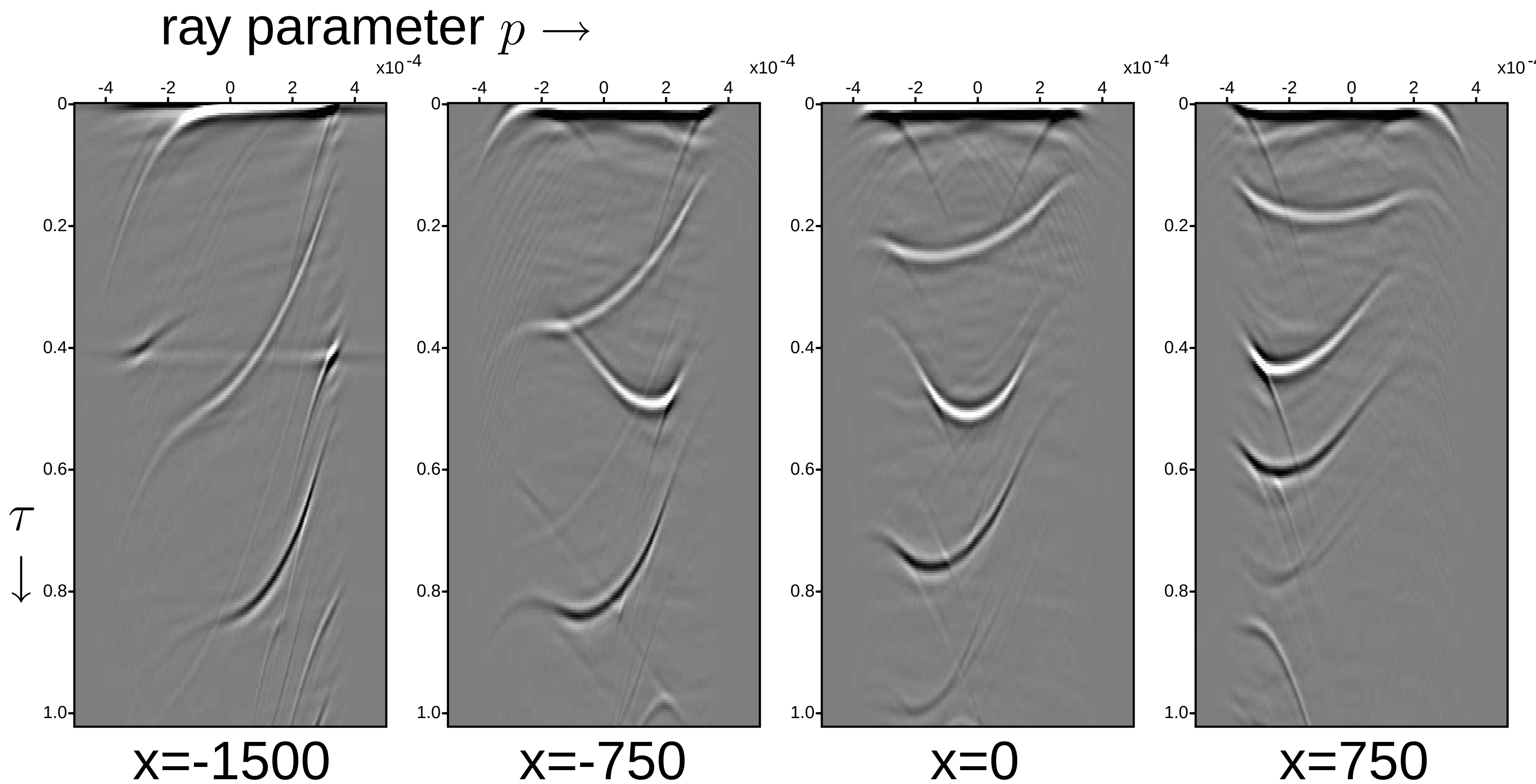
Multi layer simple 2D



Modelled Transmission response

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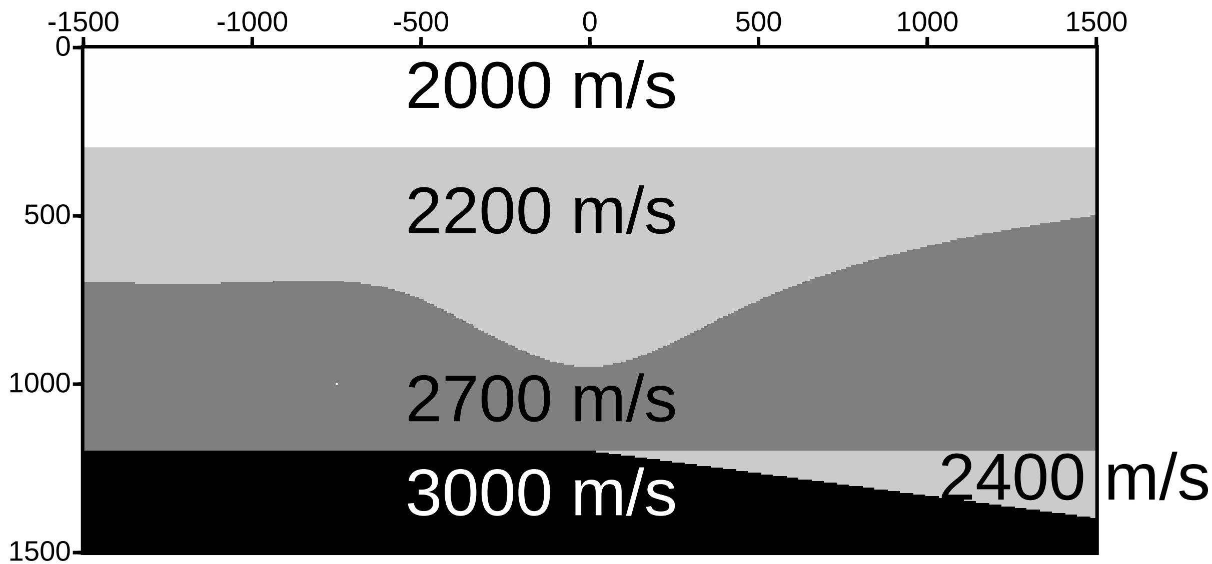
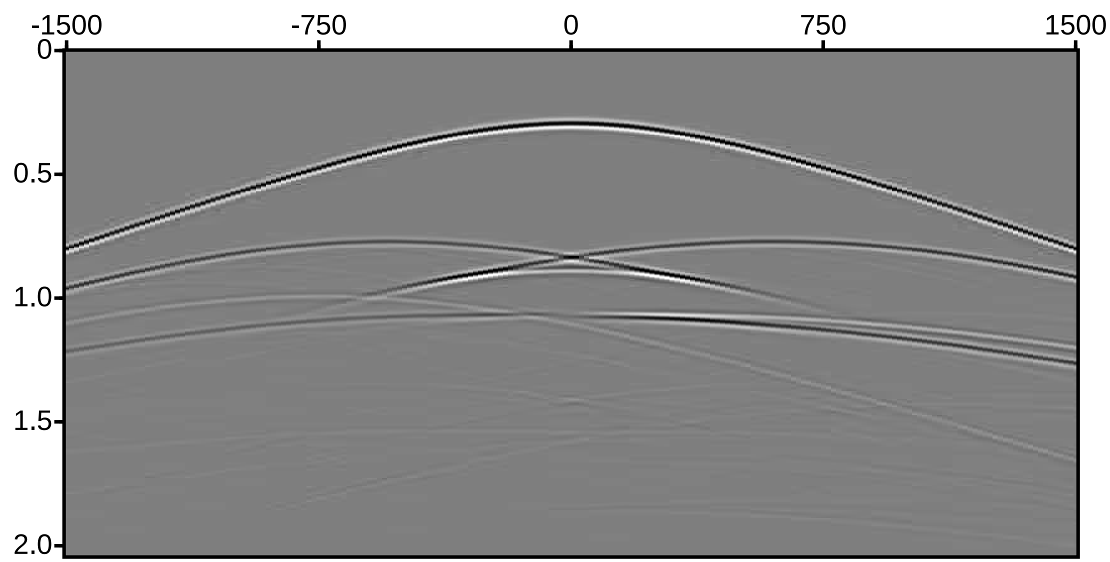
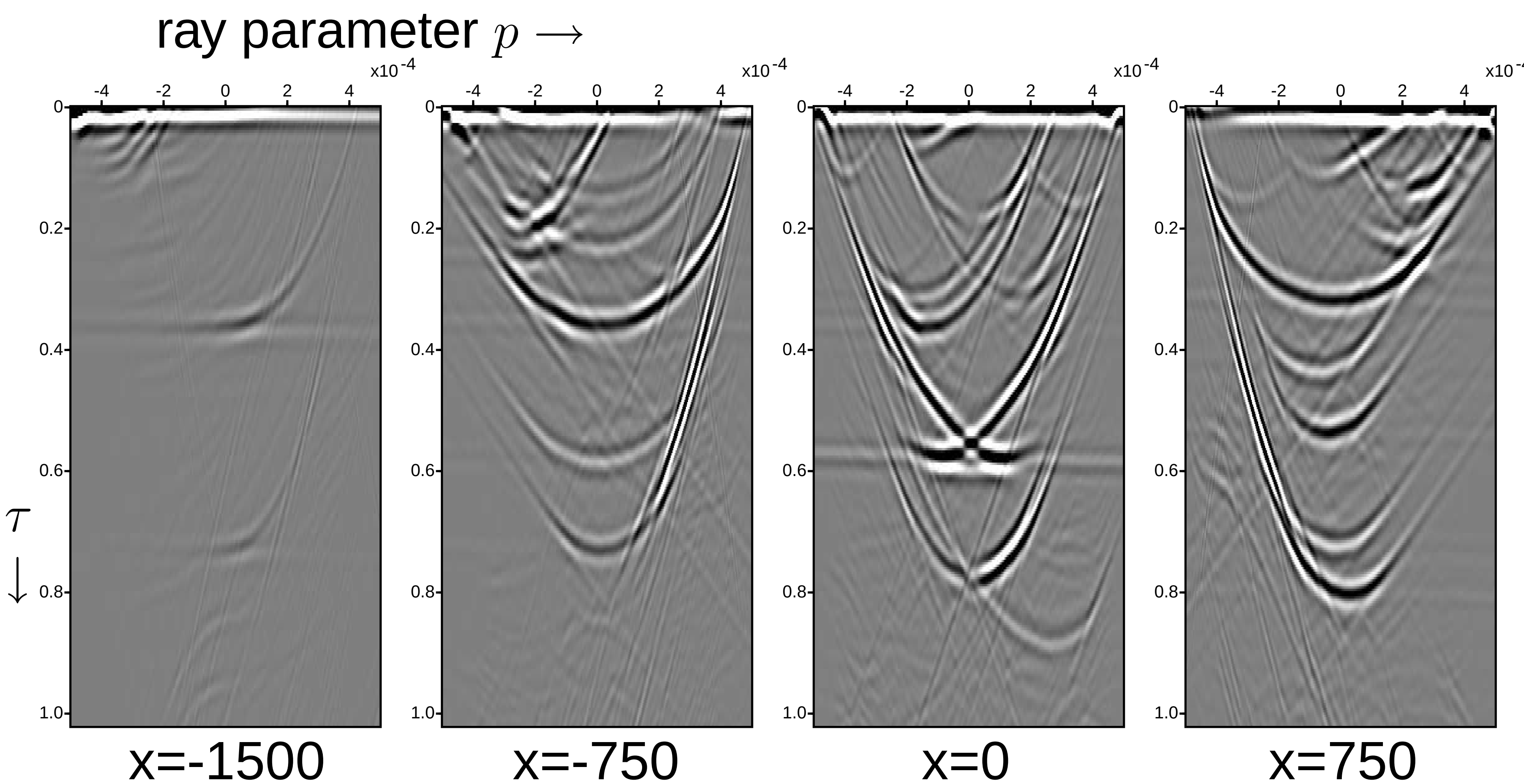
Multi layer 2D





shot record at x=0

Multi layer 2D



Calculated Eigenvalues using local 1D assumption

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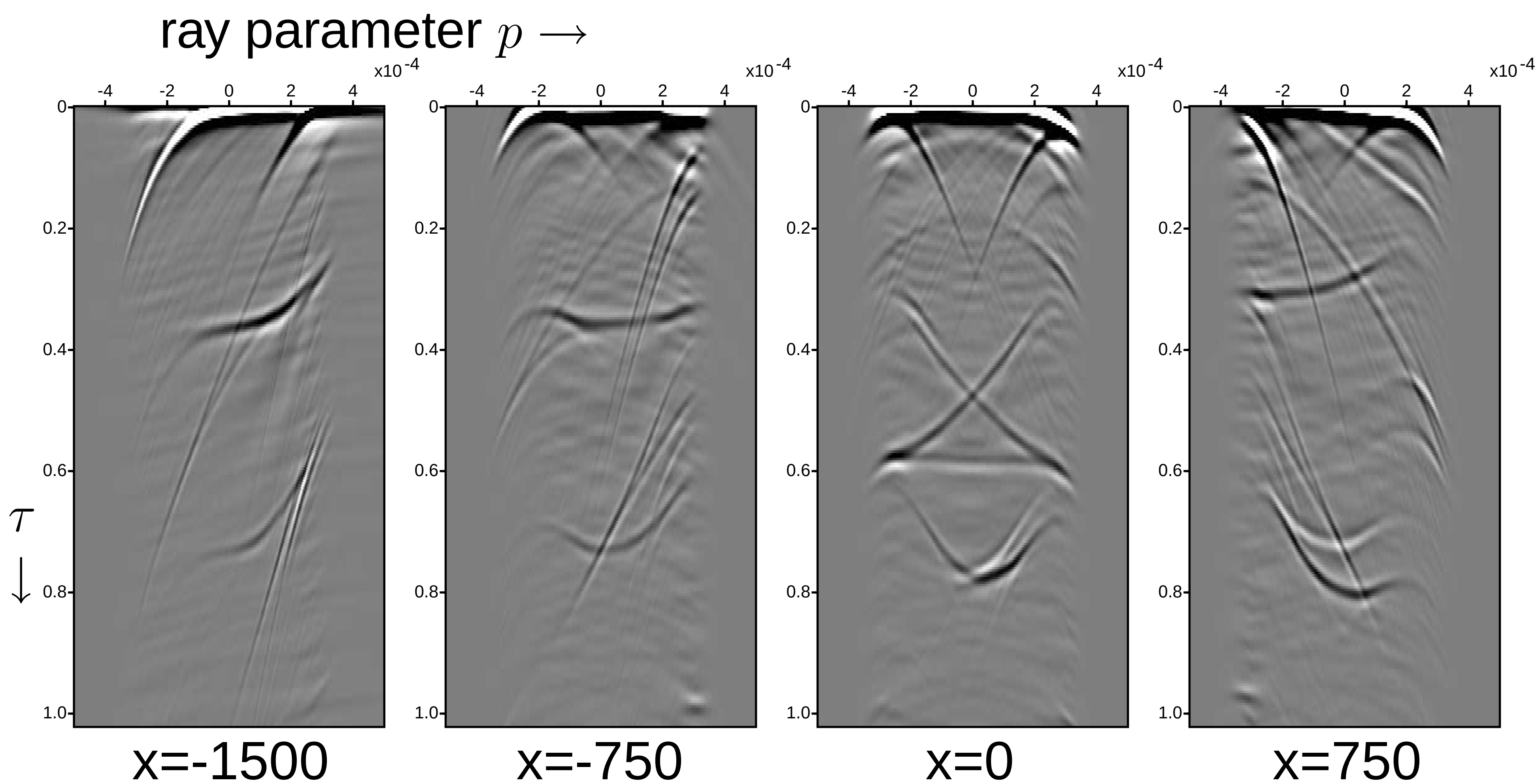
Multi layer 2D



Modelled Transmission response

Conclusions

- Based on the one-way reciprocity theorem of the correlation type one can derive:
  - explicit relation for reflectivity from passive transmission data,
  - implicit relation for transmission from active reflection data.
- Correlated reflection panels contain information of the transmission coda.
- For 1D media this coda can be extracted
- For more complex media a local 1D assumption can be used to extract an first estimate of the coda

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Acknowledgements

We would like to thank the research school ISES for supporting this research.

Downloads

Articles referred in this presentation:
http://www.xs4all.nl/~janth/Publications.html

This presentation can be found at:
http://www.xs4all.nl//~janth/Presentations/EAGE2006.pdf

For questions or remarks: janth@xs4all.nl

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