Optimum extrapolation operators: a comparison

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Introduction

In homogeneous media the one-way extrapolation operator in the k_x-ω (wavenumber-frequency) domain is a simple analytical function. The advantage of computation in the k_x-ω domain is that the desired result is obtained by *multiplication* of the data with the operator. To allow laterally varying medium functions a *convolutional* operator in the x-ω (space-frequency) domain should be used. When the spatial extrapolation operator is used in an explicit recursive depth migration algorithm it must be calculated in an optimum way to obtain reliable and stable results (Berkhout, 1982).

There are several ways to obtain a spatial convolution operator. For homogeneous media one usually starts with the exact analytical expression in the k_x-ω domain and transforms this operator back to the spatial domain. In recent years many methods have been developed to do this transformation in an efficient and optimum way. For the one-way extrapolation operator Holberg (1988), Blacquiere (1989), Hale (1990) and Nautiyal et al. (1993) have proposed methods to arrive at spatial operators which are unconditionally stable in a recursive extrapolation scheme. In this paper an alternative method is presented for an efficient and controlled transformation back to the spatial domain. The proposed method is compared with other numerical optimization methods and in the poster presentation some results of recursive depth migration, in inhomogeneous acoustic media with the different optimized extrapolation operators, are given.

Weighted least squares optimization

The most simple way to obtain space-frequency operators is to transform the exact operators in the wavenumber-frequency domain numerically back to the space-frequency domain. But this simple solution is not very efficient because the spatial convolution operator obtained in this way must be very long to give stable and accurate results. What we are looking for is a short extrapolation operator with a wavenumber-frequency spectrum which is, over a desired wavenumber band, equal or close to the exact formulation in the k_x-ω domain. This problem can be written as a matrix equation which is given by

$$\vec{\tilde{Y}} = \boldsymbol{F}\vec{Y} \ \text{, or } \tilde{Y}(n\Delta k_x) = \Delta x \sum_{M_1}^{M_2} Y(m\Delta x)\exp\left(j\, m\Delta x\, n\Delta k_x\right) \tag{1}$$

with $m = M_1, ..., M_2$ the length of the desired short operator and $n = N_1, ..., N_2$ the length of the Fourier transformation and $(M_2 - M_1) < (N_2 - N_1)$. A weighted error function $\tilde{\varepsilon}$ is defined by

$$\tilde{\varepsilon} = \vec{\tilde{E}}^h \tilde{\boldsymbol{\Lambda}} \vec{\tilde{E}} \ \text{ and } \ \vec{\tilde{E}} = \boldsymbol{F} < \vec{Y} > - \vec{\tilde{Y}} \tag{2}$$

with $\tilde{\boldsymbol{\Lambda}}$ a diagonal matrix containing a weighting function on its diagonal. By introducing this weighting function we have a good control over the desired functionality of the space-frequency operators. The least squares solution of equation (2) is given by

$$< \vec{Y} > = \left[\boldsymbol{F}^h\, \tilde{\boldsymbol{\Lambda}}\, \boldsymbol{F}\right]^{-1} \boldsymbol{F}^h\, \tilde{\boldsymbol{\Lambda}}\, \vec{\tilde{Y}} \tag{3}$$

The matrix $\boldsymbol{F}^h\, \tilde{\boldsymbol{\Lambda}}\, \boldsymbol{F}$ can be inverted fast because of its Toeplitz structure.

Comparison

Five methods for obtaining extrapolation operators wil be compared with each other (1) Truncation (inverse FFT) (2) Gaussian filtering (Nautiyal et al.,1993) (3) Smoothed Phase (Blacquiere,1989) (4) Constrained Non-linear Optimization (Holberg, 1988) (5) Weighted Least Squares (this paper). To compare the different methods homogeneous shot record migration experiments are carried out in a medium with a velocity of other traces are filled with zero's). The source wavelet is sampled with $4ms$ and has an amplitude spectrum up to 60 Hz. In Figure 1 ten depth sections are shown for the five different extrapolation operators and two different operator lengths. The truncation method is unstable due to wavenumber amplitudes greater than one. Gaussian filtering is stable but has an unacceptable amplitude decay at high propagation angles. The smoothed phase method is stable but contains artefacts at the higher wavenumbers. The non-linear optimization method is accurate and stable but it takes a very long time to compute the operators. Finally the weighted least squares method is stable, accurate and it takes only a short time to compute the operators.

Conclusions

The operators obtained with the weighted least squares optimization procedure are economic, stable and accurate. Comparing the weighted least squares operators with the computationally intensive non-linear optimized operators, it may be concluded that only a small difference occurs at the high angles: the weighted least squares operator decays a little in amplitude whereas the non-linear operators give rise to small artefacts.

References

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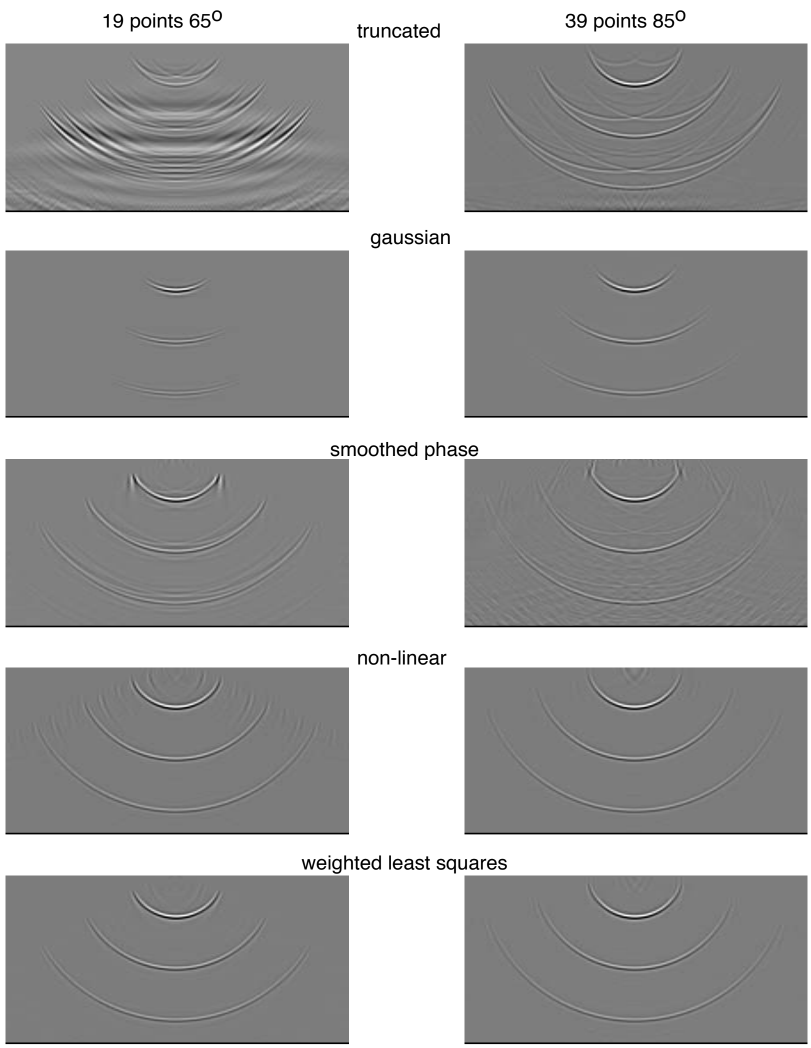

Figure1: *Imulse response for five different extrapolation operators. The left picture shows the result for an operator with 19 points and a maximum design angle of* 65^o *and the right picture shows an operator of 39 points with a maximum design angle of* 85^o.