3-D recursive extrapolation operators: a comparison

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Introduction

To visualize the 3-D subsurface of the earth, 3-D migration algorithms are needed that give accurate results within a reasonable computation time. The backbone of every recursive depth migration algorithm is a 3-D extrapolation algorithm. Extrapolation through 3-Dimensional inhomogeneous media is a computation intensive operation and requires a space-variant spatial convolution. Recently various authors (Holberg (1988), Blacquière (1989), Hale (1991), Soubaras (1992), Sollid and Arntsen (1994)) have published articles which pay attention to an optimized calculation and efficient implementation of 3-Dimensional extrapolation operators in recursive depth migration. This paper will give an overview of the existing methods and introduces several efficient optimization and implementation methods that have not yet been discussed in the geophysical literature. The computation times of the different algorithms are compared with each other and the performance of the extrapolation algorithm is checked with the aid of an example that will be shown in the presentation of the paper.

Expansions of the phase shift operator

The 3-D extrapolation algorithm that is used in recursive depth migration can be implemented in several ways. In this paper the direct method, the McClellan transformation and two series expansion methods will be discussed. These different approximations to the phase shift operator can be summarized in the following equations

$$\tilde{F}_0(k_x,k_y) = \exp\left(-jk_z\Delta z\right) \tag{1}$$

$$\approx \sum_{m=0}^{M}\sum_{n=0}^{N} F_{mn}\cos\left(k_x m\Delta x\right)\cos\left(k_y n\Delta y\right), \tag{2}$$

$$\tilde{F}_0(k_x,k_y) \approx \sum_{m=0}^{M} F_m T_m(\cos\left(\sqrt{k_x^2+k_y^2}\Delta x\right)) \tag{3}$$

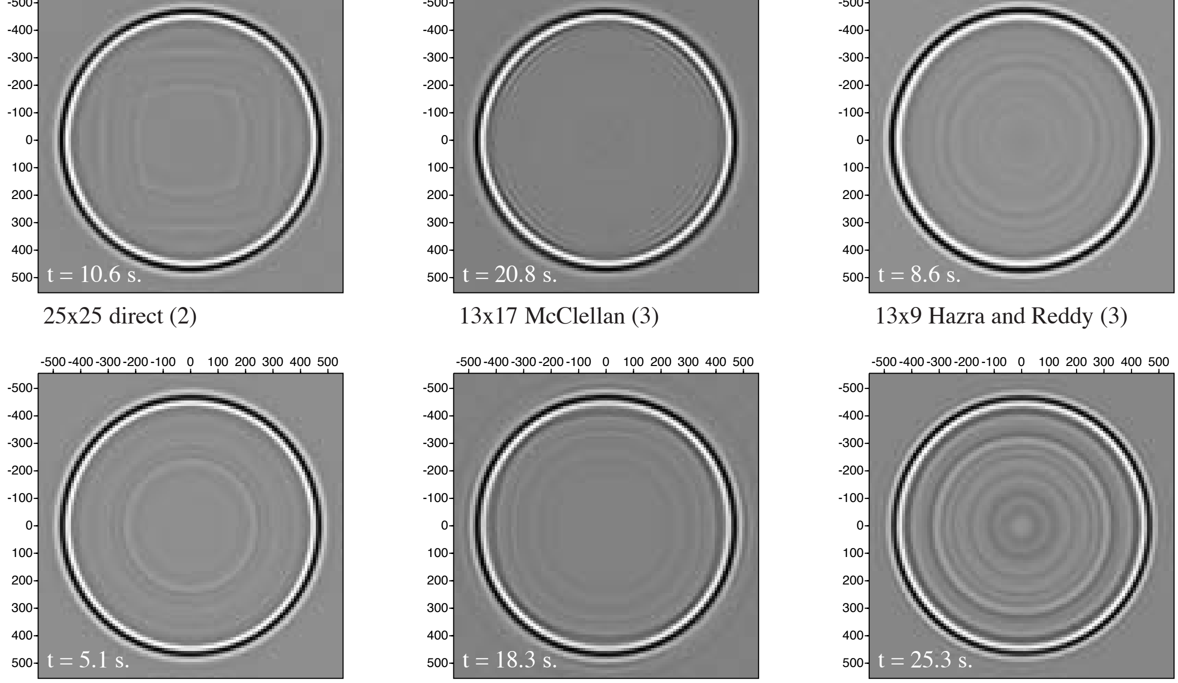
$$\approx \sum_{m=0}^{M} a_m \cos^m\left(\sqrt{k_x^2+k_y^2}\Delta x\right), \tag{4}$$

$$\tilde{F}_0(k_x,k_y) \approx \sum_{m=0}^{M} B_m T_m(k_x^2+k_y^2) \tag{5}$$

$$\approx \sum_{m=0}^{M} b_m (k_x^2+k_y^2)^m. \tag{6}$$

Equation (2) represents the direct method. The direct method uses a 2-Dimensional convolution operator. The weighted least-squares optimization method is an efficient procedure which gives stable and accurate convolution operators (Thorbecke and Rietveld, 1994). Note that this method can be further improved by a second optimization step.

Equation (3) is the McClellan approach with the Chebyshev recursion scheme. The McClellan scheme, which makes use of the 1-D optimized operator coefficients, is attractive with respect to the computation effort and by using optimized McClellan factors the accuracy for the higher angles can be further improved without much effort. The approximation of $\cos\left(k_r\Delta x\right)$ can be done with many different methods. Crucial in the performance of the extrapolation operator is that the coefficients in the expansion (Chebyshev or series) are optimized by *using* the approximation to $\cos\left(k_r\Delta x\right)$ (4). Equation (5) is the expansion in the Laplacian $k_x^2+k_y^2$ with the Chebyshev recursion scheme. Equation (6) is the series expansion in $k_x^2+k_y^2$; the use of this series expansion in recursive migration was already proposed



25x25 direct (2) | 13x17 McClellan (3) | 13x9 Hazra and Reddy (3)

13x9 series in $\cos{(k_r)}$ (4) | 15x(7+7) Chebychev with L_∞ (5) | 15x(7+7) series with L_2 (6)

Figure 1: *Depth section at 65^o for impulse responses of different methods. Note that the computation times given in the lower left corner are just an indication that differs for different machines.*

by Berkhout (1982). A disadvantage of all these expansion schemes with respect to the direct method is that it is not possible to write the algorithms in a computer ’friendly’ way due to the recursive structure in the scheme. This fact is displayed in the computation times, given in the presentation of the paper. The performance of the different extrapolation algorithms is calculated on four different systems.

In Figure 1 six depth slices are shown for six different methods. The Hazra and Reddy (1986) method is an efficient improvement of the McClellan transformation. The computation times given in the lower left corner of the pictures are calculated on a HP 9000-735 for one frequency and one depth step for a gridsize of 111x111 points. Note the large difference in computation time between the Chebychev recursion (3) and the series expansion in $\cos{(k_r \Delta x)}$ (4).

Conclusions

Taking into account the *computation time* of the different methods, the *simplicity* of the algorithms and most important the *accuracy* of the result then the **direct method** (2) is the best method for 3D extrapolation. The 2-D convolution operators should be stored in an efficient way, by using the even symmetry of the operator (one octant need be stored only), in an operator table that can be calculated in advance. If a series expansion version is used we prefer the proposed expansion in $\cos{(k_r \Delta x)}$ (4). It is our opinion that Chebychev recursion is not an advantage.

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