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## Unified Green's Function Representation for Interferometry

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### SUMMARY

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It has been shown by many authors that the cross-correlation of two acoustic wave field recordings yields the Green's function between these receivers (in geophysics this is also known as 'seismic interferometry'). Recently the theory has been extended for situations where time-reversal invariance does not hold (e.g. in attenuating media) and where source-receiver reciprocity breaks down (in moving fluids). Here we present a unified interferometric Green's function representation which captures all these situations and, due to the unified form, readily extends to more complex situations, such as electroseismic interferometry in poro-elastic media. This unified representation has applications for controlled source experiments (as in seismic exploration) as well as for passive noise recordings of uncorrelated sources in the subsurface.

## Introduction

Since the pioneering work of Weaver and Lobkis (2001), Campillo and Paul (2003) and others, the literature on retrieving the acoustic Green's function from the cross-correlation of two wave field recordings ('seismic interferometry') has expanded spectacularly. Apart from the many successful demonstrations of the method on ultrasonic, geophysical and oceanographic data, many theoretical developments have been published as well. One particular branch of theory is based on the reciprocity principle. Recent developments in this branch of research are the extension for situations where time-reversal invariance does not hold [as for electromagnetic waves in conducting media (Slob *et al.*, 2006), acoustic waves in attenuating media (Snieder, 2006a), or general scalar diffusion phenomena (Snieder, 2006b)], as well as for situations where source-receiver reciprocity breaks down [as in moving fluids (Wapenaar, 2006b; Godin, 2006)]. Recently we developed a unified representation of Green's functions in terms of cross-correlations that covers all these cases (Wapenaar *et al.*, 2006; Snieder *et al.*, 2006). Due to the unified formulation, the theory readily extends to more complex situations, such as electroseismic Green's function retrieval in poro-elastic media. In this paper we discuss the main aspects of the unified representation and discuss its physical interpretation.

## General matrix-vector equation

Diffusion, flow and wave phenomena can each be captured by the following differential equation in matrix-vector form (Wapenaar and Fokkema, 2004),

$$\mathbf{A} \frac{D\mathbf{u}}{Dt} + \mathbf{B}\mathbf{u} + \mathbf{D}_x\mathbf{u} = \mathbf{s}, \quad (1)$$

where  $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$  is a vector containing space- and time-dependent field quantities,  $\mathbf{s} = \mathbf{s}(\mathbf{x}, t)$  is a source vector,  $\mathbf{A} = \mathbf{A}(\mathbf{x})$  and  $\mathbf{B} = \mathbf{B}(\mathbf{x})$  are matrices containing space-dependent material parameters and  $\mathbf{D}_x$  is a matrix containing the spatial differential operators  $\partial_1$ ,  $\partial_2$  and  $\partial_3$ .  $D/Dt$  denotes the material time derivative, defined as  $D/Dt = \partial/\partial t + \mathbf{v}^0 \cdot \nabla$ , where  $\partial/\partial t$  is the time derivative in the reference frame and  $\mathbf{v}^0 = \mathbf{v}^0(\mathbf{x})$  the space-dependent flow velocity of the material; the term  $\mathbf{v}^0 \cdot \nabla$  vanishes in non-moving media. In the following we discuss equation (1) for some specific situations.

For acoustic wave propagation in a moving attenuating fluid we have

$$\mathbf{u} = \begin{pmatrix} p \\ v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{s} = \begin{pmatrix} q \\ f_1 \\ f_2 \\ f_3 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} \kappa & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 0 & \rho & 0 \\ 0 & 0 & 0 & \rho \end{pmatrix}, \mathbf{B} = \begin{pmatrix} b^p & 0 & 0 & 0 \\ 0 & b^v & 0 & 0 \\ 0 & 0 & b^v & 0 \\ 0 & 0 & 0 & b^v \end{pmatrix}, \mathbf{D}_x = \begin{pmatrix} 0 & \partial_1 & \partial_2 & \partial_3 \\ \partial_1 & 0 & 0 & 0 \\ \partial_2 & 0 & 0 & 0 \\ \partial_3 & 0 & 0 & 0 \end{pmatrix},$$

with  $p$  the acoustic pressure,  $v_i$  the particle velocity,  $q$  the volume injection rate,  $f_i$  the external force,  $\kappa$  the compressibility,  $\rho$  the mass density, and  $b^p$  and  $b^v$  the loss terms. Replacing  $\mathbf{A}$  by  $\rho \text{diag}(1, 0, 0, 0)$  and  $\mathbf{B}$  by  $\frac{1}{\rho\mathcal{D}} \text{diag}(0, 1, 1, 1)$  (with  $\mathcal{D}$  the diffusion coefficient), equation (1) turns into a diffusion equation.

For electroseismic wave propagation in a non-moving ( $\mathbf{v}^0 = \mathbf{0}$ ) saturated porous solid we have (Pride, 1994)

$$\mathbf{u}^T = (\mathbf{E}^T, \mathbf{H}^T, \{\mathbf{v}^s\}^T, -\boldsymbol{\tau}_1^T, -\boldsymbol{\tau}_2^T, -\boldsymbol{\tau}_3^T, \mathbf{w}^T, p^f), \quad (2)$$

$$\mathbf{s}^T = (-\{\mathbf{J}^e\}^T, -\{\mathbf{J}^m\}^T, \mathbf{f}^T, \mathbf{h}_1^T, \mathbf{h}_2^T, \mathbf{h}_3^T, \{\mathbf{f}^f\}^T, h^f) \quad (3)$$

(superscript  $T$  denoting transposition and superscripts  $s$  and  $f$  referring to the solid and fluid phase, respectively), where  $\mathbf{E}$  and  $\mathbf{H}$  are the average electric and magnetic field vectors,  $\mathbf{v}^s$  and  $\boldsymbol{\tau}_i$  the solid particle velocity and bulk traction vectors,  $\mathbf{w} = \varphi(\mathbf{v}^f - \mathbf{v}^s)$  the filtration velocity (with  $\varphi$  the porosity),  $p^f$  the fluid pressure,  $\mathbf{J}^e$  and  $\mathbf{J}^m$  the external electric and magnetic current density vectors,  $\mathbf{f}$  and  $\mathbf{f}^f$  the external forces on the bulk and on the fluid,  $\mathbf{h}_i$  and  $h^f$  the modified external deformation rates for the bulk and the fluid, and  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{D}_x$  being  $22 \times 22$  matrices.

Omitting  $\mathbf{E}$ ,  $\mathbf{H}$ ,  $\mathbf{J}^e$  and  $\mathbf{J}^m$  from  $\mathbf{u}$  and  $\mathbf{s}$  in equations (2) and (3) gives the field and source vectors for the Biot theory (Biot, 1956). Omitting in addition  $\mathbf{w}$ ,  $p^f$ ,  $\mathbf{f}^f$  and  $h^f$  gives the field and source vectors for elastodynamic wave propagation in a solid. On the other hand, omitting  $\mathbf{v}^s$ ,  $\boldsymbol{\tau}_i$ ,  $\mathbf{w}$ ,  $p^f$ ,  $\mathbf{f}$ ,  $\mathbf{h}_i$ ,  $\mathbf{f}^f$  and  $h^f$  from  $\mathbf{u}$  and  $\mathbf{s}$  in equations (2) and (3) gives the field and source vectors for electromagnetic wave propagation and/or diffusion in matter.

In all cases, matrices  $\mathbf{A}(\mathbf{x})$  and  $\mathbf{B}(\mathbf{x})$  can be replaced by temporal convolutional operators  $\mathbf{A}(\mathbf{x}, t)*$  and  $\mathbf{B}(\mathbf{x}, t)*$  to account for more general attenuation mechanisms. We define the Fourier transform of a time-dependent function  $f(t)$  as  $\hat{f}(\omega) = \int f(t) \exp(-j\omega t) dt$ , where  $j$  is the imaginary unit and  $\omega$  denotes the angular frequency. Applying the Fourier transform to all terms in the matrix-vector equation (with  $\mathbf{A}$  and  $\mathbf{B}$  defined as temporal convolutional operators) yields

$$\hat{\mathbf{A}}(j\omega + \mathbf{v}^0 \cdot \nabla) \hat{\mathbf{u}} + \hat{\mathbf{B}}\hat{\mathbf{u}} + \mathbf{D}_x\hat{\mathbf{u}} = \hat{\mathbf{s}}. \quad (4)$$

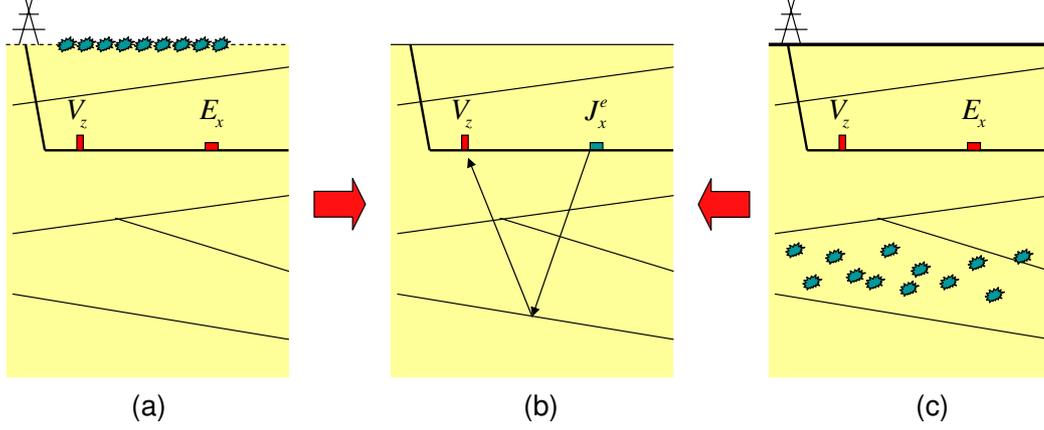


Figure 1: From (a) to (b): interferometry with controlled sources at the Earth's surface (Application 1). From (c) to (b): interferometry with uncorrelated noise sources in the Earth's subsurface (Application 3). In both examples the vertical particle velocity is cross-correlated with the horizontal electric field, yielding the electroseismic response of a horizontal electric current source observed by a vertical geophone.

### Green's matrix

For the introduction of the Green's matrix we replace the space- and frequency-dependent  $L \times 1$  source vector  $\hat{\mathbf{s}}(\mathbf{x}, \omega)$  in equation (4) by a  $L \times L$  frequency-independent point source matrix  $\mathbf{I}\delta(\mathbf{x} - \mathbf{x}_A)$ , where  $\mathbf{I}$  is the identity matrix and  $\mathbf{x}_A$  denotes the source point. Correspondingly, the  $L \times 1$  field vector  $\hat{\mathbf{u}}(\mathbf{x}, \omega)$  is replaced by a  $L \times L$  Green's matrix  $\hat{\mathbf{G}}(\mathbf{x}, \mathbf{x}_A, \omega)$ . For example, the acoustic Green's matrix is given by

$$\hat{\mathbf{G}}(\mathbf{x}, \mathbf{x}_A, \omega) = \begin{pmatrix} \hat{G}_1^{p,q} & \hat{G}_1^{p,f} & \hat{G}_1^{p,2} & \hat{G}_1^{p,3} \\ \hat{G}_1^{v,q} & \hat{G}_1^{v,f} & \hat{G}_1^{v,2} & \hat{G}_1^{v,3} \\ \hat{G}_2^{v,q} & \hat{G}_2^{v,f} & \hat{G}_2^{v,2} & \hat{G}_2^{v,3} \\ \hat{G}_3^{v,q} & \hat{G}_3^{v,f} & \hat{G}_3^{v,2} & \hat{G}_3^{v,3} \end{pmatrix}(\mathbf{x}, \mathbf{x}_A, \omega). \quad (5)$$

The superscripts refer to the type of observed wave field at  $\mathbf{x}$  and the source type at  $\mathbf{x}_A$ , respectively; the subscripts denote the different components. The  $(k, l)$ -element of  $\hat{\mathbf{G}}$  represents a receiver-type corresponding to the  $k$ th element of field vector  $\hat{\mathbf{u}}$  and a source-type corresponding to the  $l$ th element of source vector  $\hat{\mathbf{s}}$ .

### Unified Green's function retrieval by cross-correlation

Consider an arbitrary spatial domain  $\mathbb{D}$  with boundary  $\partial\mathbb{D}$  and outward pointing normal vector  $\mathbf{n} = (n_1, n_2, n_3)$  and define two points  $\mathbf{x}_A$  and  $\mathbf{x}_B$  both in  $\mathbb{D}$ . The general expression for Green's function retrieval by cross-correlation reads

$$\hat{\mathbf{G}}(\mathbf{x}_B, \mathbf{x}_A, \omega) + \hat{\mathbf{G}}^\dagger(\mathbf{x}_A, \mathbf{x}_B, \omega) = - \oint_{\partial\mathbb{D}} \hat{\mathbf{G}}(\mathbf{x}_B, \mathbf{x}, \omega) \hat{\mathbf{M}}_5 \hat{\mathbf{G}}^\dagger(\mathbf{x}_A, \mathbf{x}, \omega) d^2\mathbf{x} + \int_{\mathbb{D}} \hat{\mathbf{G}}(\mathbf{x}_B, \mathbf{x}, \omega) \hat{\mathbf{M}}_6 \hat{\mathbf{G}}^\dagger(\mathbf{x}_A, \mathbf{x}, \omega) d^3\mathbf{x} \quad (6)$$

(Wapenaar *et al.*, 2006), with  $\hat{\mathbf{M}}_5 = \mathbf{N}_\mathbf{x} + \hat{\mathbf{A}}^\dagger(\mathbf{v}^0 \cdot \mathbf{n})$  and  $\hat{\mathbf{M}}_6 = -(\underline{\nabla} \cdot \mathbf{v}^0 - j\omega)2j\Im(\hat{\mathbf{A}}) + \hat{\mathbf{B}} + \hat{\mathbf{B}}^\dagger$ , where  $\underline{\nabla}$  acts on the quantity left of it, superscript  $\dagger$  denotes transposition and complex conjugation,  $\Im$  denotes the imaginary part, and  $\mathbf{N}_\mathbf{x}$  is a matrix defined similar as  $\mathbf{D}_\mathbf{x}$ , but with  $\partial_i$  replaced by  $n_i$ . Note that  $\Im(\hat{\mathbf{A}})$  and  $\hat{\mathbf{B}} + \hat{\mathbf{B}}^\dagger$  account for the attenuation of the medium. Equation (6) is a general representation of the Green's matrix between  $\mathbf{x}_A$  and  $\mathbf{x}_B$  in terms of cross-correlations of observed fields at  $\mathbf{x}_A$  and  $\mathbf{x}_B$  due to sources at  $\mathbf{x}$  on the boundary  $\partial\mathbb{D}$  as well as in the domain  $\mathbb{D}$ . The inverse Fourier transform of the left-hand side is  $\mathbf{G}(\mathbf{x}_B, \mathbf{x}_A, t) + \mathbf{G}^T(\mathbf{x}_A, \mathbf{x}_B, -t)$ , from which  $\mathbf{G}(\mathbf{x}_B, \mathbf{x}_A, t)$  is obtained by taking the causal part.

The application of equation (6) requires independent measurements of the impulse responses of different types of sources at all  $\mathbf{x} \in \mathbb{D} \cup \partial\mathbb{D}$ . In the following we consider three special situations.

### Application 1. Controlled sources on $\partial\mathbb{D}$

Assuming the medium is lossless throughout  $\mathbb{D}$ , the domain integral in equation (6) vanishes. When the medium is non-flowing, the term  $\hat{\mathbf{M}}_5$  reduces to  $\mathbf{N}_x$ , hence

$$\hat{\mathbf{G}}(\mathbf{x}_B, \mathbf{x}_A, \omega) + \hat{\mathbf{G}}^\dagger(\mathbf{x}_A, \mathbf{x}_B, \omega) = - \oint_{\partial\mathbb{D}} \hat{\mathbf{G}}(\mathbf{x}_B, \mathbf{x}, \omega) \mathbf{N}_x \hat{\mathbf{G}}^\dagger(\mathbf{x}_A, \mathbf{x}, \omega) d^2\mathbf{x}. \quad (7)$$

In practical situations the sources are not available on a closed surface. Assuming the medium is ‘sufficiently inhomogeneous’, the closed surface  $\partial\mathbb{D}$  can be replaced by an open surface (Wapenaar, 2006a). Hence, in exploration seismology (Figure 1a) it is under specific conditions sufficient to have sources at the Earth’s surface only. Consider for example the (9, 1)-element of the electroseismic Green’s matrix  $\mathbf{G}(\mathbf{x}_B, \mathbf{x}_A, t)$ , which is the vertical particle velocity of the solid phase at  $\mathbf{x}_B$  due to an impulsive horizontal electric current source at  $\mathbf{x}_A$ , see Figure 1b. According to equation (7) this particular element is obtained by cross-correlating the vertical particle velocity at  $\mathbf{x}_B$  [the 9th row of  $\mathbf{G}(\mathbf{x}_B, \mathbf{x}, t)$ ], with the horizontal electric field at  $\mathbf{x}_A$  [the first column of  $\mathbf{N}_x \mathbf{G}^T(\mathbf{x}_A, \mathbf{x}, t)$ ], summing over the different source types at  $\mathbf{x}$  (the row-column multiplication) and integrating along all available sources on the surface  $\partial\mathbb{D}$ .

In the following we modify the right-hand side of equation (6) into a direct cross-correlation (i.e., without the integrals) of diffuse field observations at  $\mathbf{x}_A$  and  $\mathbf{x}_B$ , the diffusivity being due to a distribution of uncorrelated noise sources. Following Snieder (2006a) we separately consider the situation for uncorrelated sources in  $\mathbb{D}$  and on  $\partial\mathbb{D}$ .

### Application 2. Uncorrelated sources in $\mathbb{D}$

The boundary integral vanishes when homogeneous boundary conditions apply at  $\partial\mathbb{D}$  or, in case of infinite  $\mathbb{D}$ , when one or more elements of the loss matrices  $\Im(\hat{\mathbf{A}})$  or  $\hat{\mathbf{B}} + \hat{\mathbf{B}}^\dagger$  are non-zero throughout space. For these situations we consider a noise distribution  $\hat{\mathbf{s}}(\mathbf{x}, \omega)$  throughout  $\mathbb{D}$  (see Figure 2), where  $\hat{\mathbf{s}}$  is a vector with elements  $\hat{s}_k$ . We assume that two noise sources  $\hat{s}_k(\mathbf{x}, \omega)$  and  $\hat{s}_l(\mathbf{x}', \omega)$  are mutually uncorrelated for any  $k \neq l$  and  $\mathbf{x} \neq \mathbf{x}'$  in  $\mathbb{D}$ , and that their power spectra are the same for all  $\mathbf{x}$  and  $k$ , apart from a space- and frequency dependent excitation function. Hence, we assume that these noise sources obey the relation  $\langle \hat{\mathbf{s}}(\mathbf{x}', \omega) \hat{\mathbf{s}}^\dagger(\mathbf{x}, \omega) \rangle = \hat{\boldsymbol{\lambda}}(\mathbf{x}, \omega) \delta(\mathbf{x} - \mathbf{x}') \hat{S}(\omega)$ , where  $\langle \cdot \rangle$  denotes a spatial ensemble average,  $\hat{S}(\omega)$  the power spectrum of the noise, and  $\hat{\boldsymbol{\lambda}}(\mathbf{x}, \omega)$  is a diagonal matrix containing the excitation functions. We express the observed field vector at  $\mathbf{x}_A$  as  $\hat{\mathbf{u}}^{\text{obs}}(\mathbf{x}_A, \omega) = \int_{\mathbb{D}} \hat{\mathbf{G}}(\mathbf{x}_A, \mathbf{x}, \omega) \hat{\mathbf{s}}(\mathbf{x}, \omega) d^3\mathbf{x}$  [and a similar expression for  $\hat{\mathbf{u}}^{\text{obs}}(\mathbf{x}_B, \omega)$ ]. Evaluating the cross-correlation of the observed fields yields

$$\langle \hat{\mathbf{u}}^{\text{obs}}(\mathbf{x}_B, \omega) \{ \hat{\mathbf{u}}^{\text{obs}}(\mathbf{x}_A, \omega) \}^\dagger \rangle = \int_{\mathbb{D}} \hat{\mathbf{G}}(\mathbf{x}_B, \mathbf{x}, \omega) \hat{\boldsymbol{\lambda}}(\mathbf{x}, \omega) \hat{\mathbf{G}}^\dagger(\mathbf{x}_A, \mathbf{x}, \omega) \hat{S}(\omega) d^3\mathbf{x}. \quad (8)$$

Comparing this with the right-hand side of equation (6) (with vanishing boundary integral), we obtain

$$\{ \hat{\mathbf{G}}(\mathbf{x}_B, \mathbf{x}_A, \omega) + \hat{\mathbf{G}}^\dagger(\mathbf{x}_A, \mathbf{x}_B, \omega) \} \hat{S}(\omega) = \langle \hat{\mathbf{u}}^{\text{obs}}(\mathbf{x}_B, \omega) \{ \hat{\mathbf{u}}^{\text{obs}}(\mathbf{x}_A, \omega) \}^\dagger \rangle, \quad (9)$$

assuming  $\hat{\boldsymbol{\lambda}}(\mathbf{x}, \omega) = \hat{\mathbf{M}}_6(\mathbf{x}, \omega)$ . Hence, for those situations in which  $\hat{\mathbf{M}}_6$  is a diagonal matrix with one or more non-zero elements (e.g. for scalar diffusion or acoustic wave propagation in an attenuating medium with either real-valued  $\hat{\mathbf{A}}$  or zero flow velocity  $\mathbf{v}^0$ , for electromagnetic diffusion and/or wave propagation in a non-moving isotropic attenuating medium and, under particular conditions, for electroseismic wave propagation in an isotropic porous medium), the Green’s matrix between  $\mathbf{x}_A$  and  $\mathbf{x}_B$  can be obtained from the cross-correlation of observations at those points, assuming that a distribution of uncorrelated noise sources is present throughout  $\mathbb{D}$ , with excitation function(s) proportional to the local loss function(s) on the diagonal of  $\hat{\mathbf{M}}_6$ . The continuous injection of energy throughout  $\mathbb{D}$  is needed to overcome the dissipation (Snieder *et al.*, 2006).

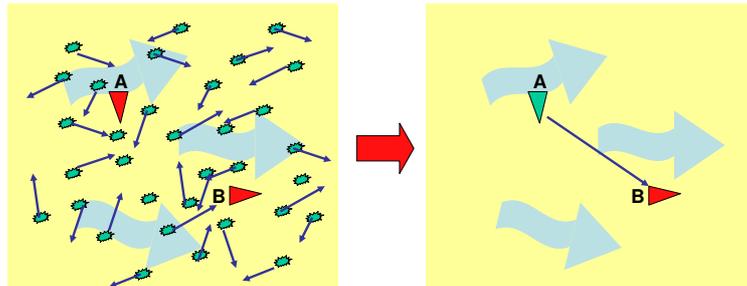


Figure 2: Interferometry with uncorrelated noise sources throughout space in a dissipative, possibly flowing, medium (Application 2). The volume distribution of sources compensates for the dissipation (Snieder *et al.*, 2006).

### Application 3. Uncorrelated sources on $\partial\mathbb{D}$

When  $\mathbb{D}$  is finite and no homogeneous boundary conditions apply at  $\partial\mathbb{D}$ , the boundary integral in equation (6) does not vanish. Assuming the losses in  $\mathbb{D}$  are small, the last integral can be ignored (see Slob *et al.* (2006) for a discussion of the effects of ignoring this integral). Hence, under this condition equation (6) implies that the Green's matrix between  $\mathbf{x}_A$  and  $\mathbf{x}_B$  can be retrieved from cross-correlations of responses of independent impulsive sources on  $\partial\mathbb{D}$  only. To make equation (6) suited for uncorrelated noise sources on  $\partial\mathbb{D}$ , matrix  $\hat{\mathbf{M}}_5$  must be 'diagonalized' so that we can follow the same procedure as above. The term  $\hat{\mathbf{A}}^\dagger(\mathbf{v}^0 \cdot \mathbf{n})$  in  $\hat{\mathbf{M}}_5$  is diagonal for scalar diffusion and for acoustic wave propagation in a flowing medium, whereas it vanishes in non-moving media. However,  $\mathbf{N}_\mathbf{x}$  is not diagonal for any of the discussed applications. Diagonalization of the integral  $-\oint_{\partial\mathbb{D}} \hat{\mathbf{G}}(\mathbf{x}_B, \mathbf{x}, \omega) \mathbf{N}_\mathbf{x} \hat{\mathbf{G}}^\dagger(\mathbf{x}_A, \mathbf{x}, \omega) d^2\mathbf{x}$  involves decomposition of the sources at  $\partial\mathbb{D}$  into sources for inward and outward propagating waves. Following the approach discussed by Wapenaar and Fokkema (2006), assuming  $\partial\mathbb{D}$  is far away from  $\mathbf{x}_A$  and  $\mathbf{x}_B$ , we may approximate the integral (including the minus sign) by  $\oint_{\partial\mathbb{D}} \hat{\mathbf{G}}^\phi(\mathbf{x}_B, \mathbf{x}, \omega) \boldsymbol{\lambda}(\mathbf{x}) \{\hat{\mathbf{G}}^\phi(\mathbf{x}_A, \mathbf{x}, \omega)\}^\dagger d^2\mathbf{x}$  + 'ghost', where 'ghost' refers to spurious events due to cross products of inward and outward propagating waves. When  $\partial\mathbb{D}$  is irregular (which is the case when the sources are randomly distributed, as in Figure 1c) these cross products do not integrate coherently and hence the spurious events are suppressed (Draganov *et al.*, 2006). When the medium at and outside  $\partial\mathbb{D}$  is homogeneous and isotropic the spurious events are absent. Superscript  $\phi$  refers to new source types at  $\mathbf{x} \in \partial\mathbb{D}$  and  $\boldsymbol{\lambda}(\mathbf{x})$  is a diagonal matrix containing normalization factors. Hence, assuming a distribution of uncorrelated noise sources  $\hat{\mathbf{s}}^\phi(\mathbf{x}, \omega)$  on  $\partial\mathbb{D}$ , we arrive in a similar way as above at equation (9), but this time with the observed field vector at  $\mathbf{x}_A$  expressed as  $\hat{\mathbf{u}}^{\text{obs}}(\mathbf{x}_A, \omega) = \oint_{\partial\mathbb{D}} \hat{\mathbf{G}}^\phi(\mathbf{x}_A, \mathbf{x}, \omega) \hat{\mathbf{s}}^\phi(\mathbf{x}, \omega) d^2\mathbf{x}$  [and a similar expression for  $\hat{\mathbf{u}}^{\text{obs}}(\mathbf{x}_B, \omega)$ ].

### Conclusion

We have developed a unified representation for interferometry, which applies to diffusion phenomena, acoustic waves in flowing attenuating media, electromagnetic diffusion and wave phenomena, elastodynamic waves in anisotropic solids and electroseismic waves in poro-elastic media. This unified representation has applications for controlled source experiments (as in seismic exploration) as well as for passive noise recordings of uncorrelated sources in the subsurface. Numerical and real data examples will be discussed in the presentation.

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