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# A Brief Summary of Hessian Based Modelling in a Shared Earth Environment

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# SUMMARY

Validating an earth model by simulating a migration image is an important step in a shared earth environment. However, the high computational cost of generating 3D synthetic data, followed by the process of migration, limits the number of scenarios that can be validated. To overcome this computational cost, a novel strategy is used where a migration image is simulated by filtering a model. A filter represents one local element of the Hessian, which is obtained by computing the image of a single unit-strength scatterer in a macro velocity model (the impulse response of the filter). One of the key properties of this approach is that the model that describes a target-zone is decoupled from the macro-velocity model that is used to compute a filter. Consequently, different models can be filtered with the same filter. In addition, migration images of, for example, a walk-away VSP dataset or the influence of seismic uncertainties can be simulated. Therefore, these filters enhance a shared earth modelling approach.



### Introduction

An earth model is used in a collaborative environment in which some members provide information for its construction and others utilize the result. Validating an earth model by simulating a migration image is an important step. To simulate seismic images for a range of geologic models, which differ only in a target zone, a convolution is performed between a source signature and each reflectivity trace derived from a geologic model. This is the industry-practice and known as the 1D convolution model. However, real data examples show that horizontal smearing, which is not simulated by the 1D convolution model result, is essential to understand the seismic expression (see for example Toxopeus et al., 2009). An alternative for the 1D convolution is to fully simulate a migration image in a complex overburden setting. However, the high computational cost of generating 3D synthetic data, followed by the process of migration, limits the number of scenarios that can be validated. Recently the 1D convolution model is extended; a migration image for a complex overburden is simulated by filtering a model with a spatial resolution filter. A filter represents one local element of the Hessian (Schuster and Hu, 2000, Lecomte et al., 2003, Lecomte, 2006, Toxopeus et al., 2008 and Toxopeus et al., 2009). In this paper, a brief summary of Hessian based modelling is presented, followed by examples based on a simple-salt model. Though the 2D examples presented show only P-wave data, the approach is valid in three dimensions for an arbitrary elastic earth model.

### Simulating a migration image by filtering a model

Synthetic seismic data d in the Fourier domain can be described as:

$$\hat{d} = \hat{L}\hat{m}(x), \quad (1)$$

where x denotes the spatial coordinate vector of position in the earth model,  $\hat{L}$  is a forward modelling operator simulating the real seismic experiment and  $\hat{m}$  is a geological model. A migration image

 $(\hat{m}_{mig})$  is obtained by using a migration operator  $(\hat{L}^{H})$ , which approximates the inverse of  $\hat{L}$ . This extends the relation to:

$$\hat{m}_{mig} = \hat{L}^{H} \{ \hat{L} \hat{m}(x) \}.$$
 (2)

 $\hat{L}^{H}\hat{L}$  is known as the Hessian  $(\hat{H})$ . One element of the Hessian is represented by a spatial resolution

filter  $(\hat{C})$ , which is obtained by computing the image of a single unit-strength scatterer in a macro velocity model (the impulse response of the filter). For a model with a background velocity of 2000 m/s and a symmetric acquisition setup, a spatial resolution filter is shown in Figure 1 (a). Using the spatial-resolution filter, a migrated image is simulated as

$$\hat{m}_{mig}(x) = \int \hat{H}(x, x')\hat{m}(x')dx', \qquad (3)$$

where  $\hat{H}(x, x') = \hat{C}\delta(x - x')$ , with x' the spatial coordinate vector of the unit-strength scatterer. Thus, instead of forward modelling and subsequently applying a migration algorithm, a migration image is simulated by filtering or in order words "blurring" a model by three-dimensional filter (Figure 2 (a)). If an infinite recording setup and a layered macro velocity are assumed, the shape of the filter collapses to approximately a single trace that models the source signature. A 1D-filter is the input to the 1D convolution model that is the industry practice to simulate migration data in a shared earth environment. Real data examples and more background information is found in Schuster and Hu, 2000, Lecomte et al., 2003, Lecomte, 2006, Toxopeus et al., 2008 and Toxopeus et al., 2009 (includes MATLAB scripts for a hands-on experience). In the same way, the inverse Hessian is used to deblur and thereby enhance the spatial resolution of a migration image (Aoki and Schuster, 2009 and Tang, 2009). However, the industry practice to estimate attributes such as acoustic impedance from a migration image is driven by a seismic inversion process known as constrained sparse-spike inversion, which is based on the 1D convolution model (see Veeken and Da Silva, 2004). To simulate the correct blurring of real sparse-spike inverted data, an inverted spatial resolution filter can be used (Toxopeus et al., 2008). A further detailed discussion is outside the scope of this paper. Finally, we



refer to the introduction of Tang (2009) for a recent overview on different methods on how to compute (elements of) the Hessian.

### Simulating a migration image in a shared earth environment

A macro velocity model is used to compute a spatial resolution filter (Figure 3 (a)). This model is obtained during velocity estimation of the real data or based on rock-physics depth-trends. A detailed reservoir model is shown in Figure 3 (b). The shapes in a model are guided by the interpretation of the real migration data or for example based on an analog.

Simulating a stacked migration image

In our examples, a spatial resolution filter is computed using a forward  $(\hat{L})$  and migration

 $(\hat{L}^{H})$  operator based on the one-way wave-equation as described by Thorbecke et al., 2004. Although the same acquisition setup and input wavelet are used, the shape of the zero-offset spatial resolution filter computed in the simple-salt model significantly differs from the filter derived in the homogenous model (Figure 1 (a) and (b)). This has an important influence on the interpretation of the simulated migration images of the reservoir (Figure 2 (a) and (b)). The shape of the filter determines which reflectors are suppressed, i.e., in this case the salt flank and which remain on a migration image. In a shared-earth environment, commonly different reservoir scenarios are tested. To enable this, the filter is re-used for the different simulations. The assumption is that the macro-velocity model is kept constant. We refer to Lecomte, 2008 and Toxopeus et al., 2008, for real-data examples and a more detailed analysis of the spatial resolution filter. In an exploration setting, commonly different seismic volumes based on different migration operators are produced. To help an interpreter to understand the effect of different operators on the reservoir model, the forward modelled data are migrated with a Kirchhoff algorithm (a 60 degree dip limitation used and no attempts have been made to optimize parameters) and a smoothed version of the simple-salt model. The Kirchhoff-filter has a different shape compared to the wave-equation filter (Figure 1 (b) and 1 (c)). In this case, the resulting image has less (maximum up to 60 degrees) dip information (Figure 2 (c)). This highlights the importance of selecting the proper operator (choice parameters) and used of a proper velocity model to compute a filter, and consequently a simulated migration image that will match the real migrated data. Finally, the shape of a filter derived in an isotropic versus anisotropic macro-velocity model is compared by Lecomte and Kaschwich, 2008.

Sensitivity analysis of seismic data

In the previous discussion, the main parameters controlling the shape of the filter and subsequently the migration image result are: acquisition parameters, overburden, and used migration-operator. However, it was tacitly assumed that all seismic processing steps were ideally performed. For example, to properly perform migration, a velocity model of the subsurface is needed, and the effect of statics has to be removed from the recorded data. In order to understand these effects on a migration image, they can be included in the Hessian. For example, the effect of focusing and defocusing due to a wrong macro-velocity in migration and statics are shown in Toxopeus et al., 2008 and Toxopeus et al., 2009, respectively. To simulate the effect of internal multiple scattering a density model with a strong contract is used and the forward modelled data of the point scatterer are computed based on a finite-difference scheme. Next, the forward modelled data are Kirchhoff migrated. Because the multiple-energy is not properly handled by the migration operator, the filter shows defocused energy (Figure 1 (d)), which introduces additional blurring in the migration image (Figure 2 (d)).

## Data integration

In shared earth modelling, different experiments are commonly performed to image a reservoir. For example, a GPR-filter is used to validate a reservoir model based on migrated GPR data of an analog. Next, the model is up-scaled and a streamer-filter is designed and a migration image of a reservoir is simulated (Toxopeus et al., 2006). Similarly, walk-away VSP data are simulated with a VSP-filter



(Figure 1 (e)). To compute a VSP-filter, forward modelled data that simulate a VSP experiment are migrated using the steps outlined in Thorbecke, 2008. The VSP migration image shows that the salt flank is imaged (Figure 2 (e)).

#### Conclusions

For the validation of a geological model in a shared earth environment, we simulate a migration image by filtering a model with a spatial resolution filter which represents one element of the Hessian. The spatial-resolution filter captures all acquisition, seismic processing, and overburden-related effects. Key is that the filter can be re-used to validate different geological scenarios, under the assumption that the macro velocity model (overburden) is constant. In addition, the Hessian can be used to visualize the sensitivity of a migrated image and the interpretation of migration images computed from different seismic experiments. Therefore, these filters enhance an iterative earth modelling approach in a shared earth environment.

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**Figure 1:** (a) A filter derived in a homogeneous model of 2000 m/s. (b)- (e) Filter derived in a macro velocity model (Figure 3 (a)): (b) a wave-equation filter, (c) a Kirchhoff filter, (e) a Kirchhoff filter including defocused multiple-energy and (e) a walk-way VSP-filter. The arrow points out operator smearing. The double arrow points out defocused energy.

#### References

Aoki, N., and G.T. Schuster, 2009. Fast least-squares migration with a deblurring filter, Geophysics 74, WCA83.

Lecomte, I., H. Gjøystdal, and Å. Drottning, 2003. Simulated prestack local imaging: a robust and efficient interpretation tool to control illumination, resolution, and time-lapse properties of reservoirs, 73<sup>rd</sup> annual SEG meeting Expanded Abstracts, 1525–1528.

Lecomte, I., 2006. Illumination, resolution, and incidence-angle in PSDM, 76th annual SEG meeting Expanded Abstracts, 2544–2548.

Lecomte, I., 2008. Resolution and illumination analyses in PSDM: a ray-based approach, The Leading Edge 27 (5). Lecomte, I., and T. Kaschwich, 2008. Closer to real earth in reservoir characterization: A 3D isotropic/anisotropic PSDM simulator, 78<sup>th</sup> annual SEG meeting Expanded Abstracts, 1570-1573.



Schuster, G. T., and J. Hu, 2000. Green's function for migration: Continuous recording geometry: Geophysics, 65, 167–175. Tang, Y., 2009. Target-oriented wave-equation least-squares migration/inversion with phase-encoded Hessian, Geophysics 74, WCA95.

Thorbecke, J., 2008. Imaging of steep flanks by focal sources, 78th annual SEG meeting Expanded Abstracts, 2391-2394.

Thorbecke, J., K. Wapenaar, and G. Swinnen, 2004. Design of one way wavefield extrapolation operators, using smooth functions in WLSQ optimization, Geophysics 69 (4), 1037–1045.

Toxopeus G., R. Donselar, T. Dreijer, S. Petsersen, S. Lambot, and E.C. Slob. Spatial resolution filter as transfer operator from modern-analogue data to reservoir model, In: 11th International Conference on Ground Penetrating Radar, Ohio State University, Columbus, USA, 19-22 June, Chi-Chih Chen and Joel T. Johnson ed(s), 2006, p. Proceedings, p. 4p.

Toxopeus, G., J. Thorbecke, K. Wapenaar, S. Petersen, E. Slob, and J. Fokkema, 2008. Simulating migrated and inverted seismic data by filtering a geologic model: Geophysics, 73, no. 2, T1–T10.

Toxopeus, G., J. Thorbecke, S. Petersen, K. Wapenaar, and E. Slob, 2009. Simulating migrated seismic data by filtering an earth model: A MATLAB® implementation, Computers and Geosciences, in press.

Veeken, P. C. H., and M. Da Silva, 2004. Seismic inversion methods and some of their constraints: First Break, 22, 47-70.



**Figure 2** (a) – (e) Simulated migration images based on a 2D convolution between Figure 3 (b) and Figures 1 (a) - (e), respectively. The single and double arrows highlight the salt flank and a reservoir structure, respectively.



*Figure 3* (a) A simple salt-model. An asterisk denotes the position of a spatial resolution filter. A structure that could trap hydrocarbons near a salt flank is shown by dashed lines. A detailed reservoir (based on the Sigsbee model) is model is shown in (b).