Seismic Processing 2: Mathematical Methods

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Surface-Related Multiple Elimination Based on Reciprocity SP2.1

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SUMMARY

The removal of surface related wave phenomena as water surface multiples in the marine case is essential for further processing of the data. In this procedure any relevant subsurface information should be maintained. The problem can be formulated by means of the Rayleigh reciprocity theorem and this leads to an integral equation of the second kind for the desired pressure field in the water layer. This integral equation has been solved numerically either directly by matrix inversion or iteratively by the Neumann series solution.

INTRODUCTION

The presence of surface related wave phenomena in geophysical data as water-surface multiples in the marine case leads to problems in further analysis of the data in inversion or migration. This removal has to be effected without changing any relevant subsurface information present in the recorded data. Fokkema and Van den Berg (1990) have shown that the Rayleigh reciprocity theorem furnishes the tool for this removal. In this theorem the interaction of two non-indentical states is considered. One state is identified with the actual situation, while the other is the desired one: the same full elastodynamic geology but without the water surface. As a result an integral equation of the second kind has been arrived at. This integral equation is solved directly by matrix inversion or iteratively by the Neumann series solution. Synthetic examples show excellent performance of the direct matrix inversion method, while the Neumann series solution does not converge.

DESIRED FIELD INTEGRAL EQUATION

In the marine case we have the situation as shown in Fig. 1. The domain of interest is the half-space $D = \{\boldsymbol{x} \in \mathbb{R}^3;\ -\infty < x,y < \infty,\ 0 < z < \infty\}$. This half-space consists of the water layer D_w and the earth geology D_g with boundary ∂D_g. The pressure at the water surface $z = 0$ is zero and the material constants of D_w are ρ_w and κ_w. As a source a point source of the volume injection type is used (air gun or water-gun) and is located at $\boldsymbol{x}^S$. The seismic response, the pressure is measured at $\boldsymbol{x}^R$ below the water surface. The analysis is carried in the frequency domain (time factor $\exp(-i\omega t)$). The pertaining acoustic field quantities at a receiver located at $\boldsymbol{x}^R$ due to a source at $\boldsymbol{x}^S$ are the acoustic pressure $p(\boldsymbol{x}^R|\boldsymbol{x}^S)$ and the particle velocity $\boldsymbol{v}(\boldsymbol{x}^R|\boldsymbol{x}^S)$.

In order to remove the effect of the water surface, Fokkema and Van den Berg (1990) have applied Rayleigh's reciprocity theorem between this actual situation and a desired situation where the waterlayer extends to $z \to -\infty$. In this desired state the acoustic pressure is given by p^d. This desired field is written as a superposition of an incident-field and a reflected-field constituent according to

$$p^d(\boldsymbol{x}^R|\boldsymbol{x}^S) = p^i(\boldsymbol{x}^R|\boldsymbol{x}^S) + p^r(\boldsymbol{x}^R|\boldsymbol{x}^S)\,. \tag{1}$$

Let us define the spatial Fourier transform with respect to the horizontal receiver coordinates as

$$\tilde{f}(\alpha,\beta,z|\boldsymbol{x}') = \iint_{-\infty}^{\infty} f(x,y,z|\boldsymbol{x}')\exp(i\alpha x + i\beta y)\,dx dy, \tag{2}$$

and with respect to the horizontal source coordinates as

$$\hat{f}(\boldsymbol{x}|\alpha,\beta,z') = \iint_{-\infty}^{\infty} f(\boldsymbol{x}|x',y',z'|)\exp(-i\alpha x' - i\beta y')\,dx'dy'. \tag{3}$$

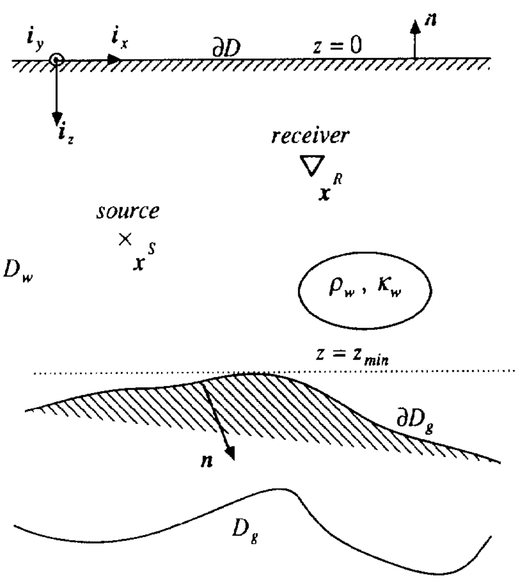

Fig. 1. The actual marine configuration.

The inverse of these transforms follow from the standard definitions. Then, a second-kind integral equation

$$a(x',y') + \iint_{-\infty}^{\infty} K(x,y|x',y')\, a(x,y)\, dx dy = b(x',y'), \qquad (x',y') \in \mathbb{R}^2, \quad (4)$$

for the desired pressure field

$$a(x,y) = p^r(\boldsymbol{x}^R|x,y,z^S) \quad (5)$$

has been obtained. The kernel K of the integral equation is obtained from the inverse Fourier transform of

$$\tilde{K}(\alpha,\beta|x',y') = \frac{-i\omega\rho_w}{S}\tilde{v}_z(\alpha,\beta,0|x',y',z^S)\exp(i\gamma_w z^S), \quad (6)$$

and the known function of the integral equation is given by

$$b(x,y) = p(\boldsymbol{x}^R|x,y,z^S) - p^i(\boldsymbol{x}^R|x,y,z^S) - \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} \tilde{p}^i(\boldsymbol{x}^R|\alpha,\beta,0)\, \frac{-i\omega\rho_w}{S}\tilde{v}_z(\alpha,\beta,0|x,y,z^S)\, d\alpha d\beta. \quad (7)$$

This known term is the deghosted field. One part of this field is the desired reflected field, while the remainder consists of contributions from reflections against the water surface $z = 0$. The latter can only be removed by solving the integral equation. The wave field quantity $a = p^r$ has to be solved for fixed $\boldsymbol{x}^R$ and variable $\boldsymbol{x}^S$ (common-receiver domain). In Eq. (7), S denotes the source signature or source wavelet related to the volume injection as

$$S = -i\omega\rho_w Q. \quad (8)$$

Note that the integration variables in the integral equation (4) are the horizontal coordinates of the various source locations of p^r, while the receiver position is fixed (common-receiver domain). Further, in the kernel and in the known function we have the expression

$$\frac{i\omega\rho_w}{S}\tilde{v}_z(\alpha,\beta,0|\boldsymbol{x}^S) = \frac{\gamma_w}{\sin(\gamma_w z^R)}\frac{\tilde{p}(\alpha,\beta,z^R;\boldsymbol{x}^S)}{S} + \begin{cases} \exp(i\alpha x^S + i\beta y^S)\dfrac{\sin\left(\gamma_w(z^R - z^S)\right)}{\sin(\gamma_w z^R)}, & 0 < z^S < z^R, \\ 0, & z^R \le z^S < z_{\min}, \end{cases} \quad (9)$$

where,

$$\gamma_w = \left(\frac{\omega^2}{c_w^2} - \alpha^2 - \beta^2\right)^{\frac{1}{2}}, \quad \mathrm{Re}\{\gamma_w\} \ge 0\,, \quad (10)$$

in which $c_w = (\rho_w \kappa_w)^{-\frac{1}{2}}$. In order to obtain the Fourier transform $\tilde{p}(\alpha,\beta,z^R|\boldsymbol{x}^S)$ we need the data for a fixed source point $\boldsymbol{x}^S$ in many receiver points (common-shot domain). Note that the in the first term of the right-hand side of Eq. (9) we need a deconvolution of the actual measured data. In this paper we assume that the source wavelet is known. In case that $\boldsymbol{x}^R$ is located at the surface $z = 0$, the expression of Eq. (9) does not hold and the particle velocity $v_z(x,y,0|\boldsymbol{x}^S)$ has to be measured. This is not applicable in the marine case. The special case that both $\boldsymbol{x}^R$ and $\boldsymbol{x}^S$ are located on $z = 0$, which is the land-seismics situation, is dealt with by Verschuur et al. (1988).

In order to simplify the discussion we write the integral equation (4) as an operator equation of the form

$$a + \mathrm{K}a = (\mathrm{I} + \mathrm{K})a = b. \quad (11)$$

The kernel of this operator equation is non-singular. Therefore we can use any integration rule to replace the integration by a discrete summation. This procedure leads to a system of linear algebraic equations for the discrete values of the desired reflected field a. An alternative way to solve Eq. (11) is based on the Neumann series solution, viz.

$$a = b - \mathrm{K}b + \mathrm{K}(\mathrm{K}b) - \cdots + (-\mathrm{K})^n b + \cdots\,. \quad (12)$$

NUMERICAL EXAMPLES

We first test our scheme for the trivial laterally-invariant configuration. We have computed two-dimensional synthetic data pertaining to a four-layer model. The source depth z^S is 7.5 m, while we employed 127 receiver positions at a depth z^R of 5 m in a split-spread configuration. The receiver spacing is 12.5 m. We have taken 127 shots with a shot increment of 12.5 m. In Fig. 2, the synthetic data of our four-layer model are presented. Fig. 2a presents the relevant data including the water surface multiples, while Fig. 2b presents the data without these multiples. Fig. 2c shows the results of our multiple elimination procedure using the matrix inversion method for the solution of the integral equation. We observe that the internal multiples are preserved in our removal procedure. We have implemented the Neumann series solution method as well, but no convergent results has been detected.

In order to simulate lateral varying data, we have computed two-dimensional synthetic data from a rigid strip of 140 m embedded in a semi-infinite waterlayer at a depth of 100 m. The computer implementation of the present forward problem is based on a conjugate-gradient iterative solution of an integral equation over the strip domain (cf. Van den Berg, 1984). The source depth z^S is 7.5 m, while we employed 127 receiver positions at a depth z^R of 5 m in a split-spread configuration. The receiver spacing is 3.5 m. We have taken 127 shots with a shot increment of 3.5 m. For the receiver location above the midpoint of the strip, these results are

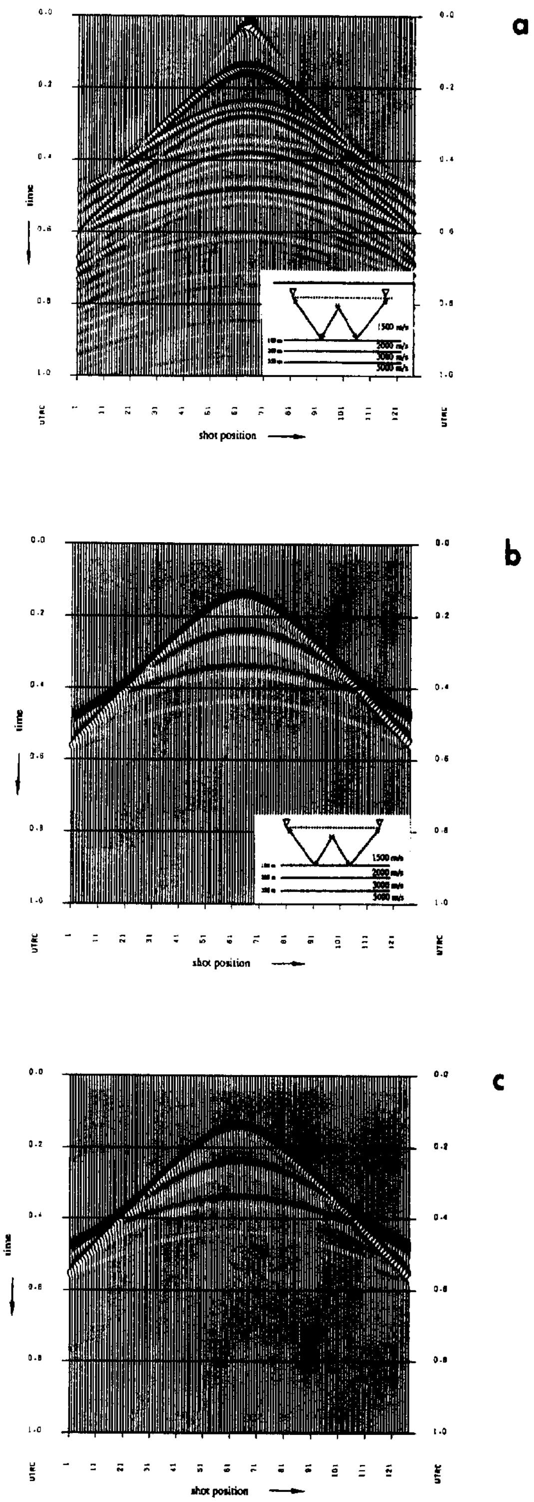

Fig. 2. Common-receiver gather of the four-layer configuration; (a) including the water surface; (b) without the water surface; (c) results of the removal procedure.

presented in Fig. 3a. The synthetic data from a rigid strip in a water embedding of infinite extent for the same receiver position are presented in Fig. 3b. The results of our multiple removal procedure using matrix inversion are presented in Fig. 3c. Note the excellent agreement with the ideal situation of Fig. 3b. The diffracted energy from the edges of the strip is preserved in our removal procedure. Again, the Neuman series solution of the integral equation did not converge in this case.

In Fig. 4, we present similar results for the receiver location above the left-hand edge of the strip. In this case, the phenomenon of missing data occurs. In the common-receiver domain we are missing 20 shot traces. This lack of data is clearly visible in the final results of our removal procedure: in Fig. 4c, we observe an artifact. Note that we have not employed any form of spatial filtering to suppress these artifacts.

CONCLUSIONS

From a proper application of the reciprocity theorem the removal of surface related wave phenomena is performed by solving an integral equation of the second kind numerically. The excellent performance of the scheme has been illustrated with synthetic examples. It has been detected that the iterative solution based on a Neumann series expansion does not lead to the desired results. Incomplete data leads to artifacts in the data resulting from our removal procedure. Future reseach will be concentrated on handling incomplete data, as missing first offsets, finite aperture and estimation of the wavelet.

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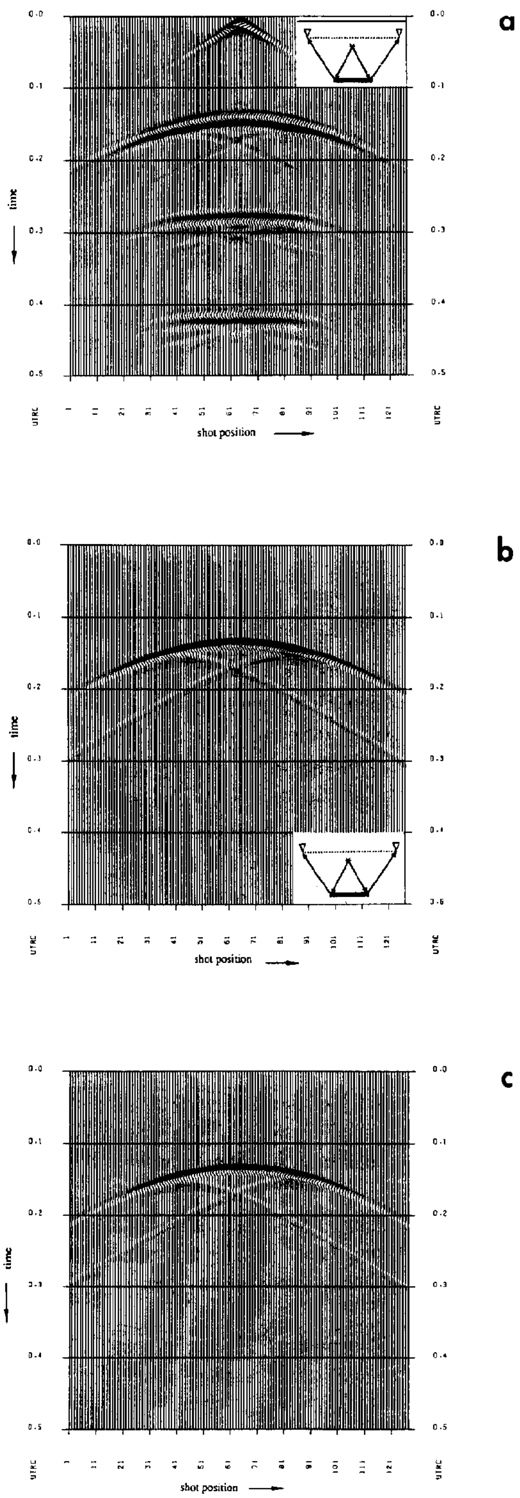

Fig. 3. Common-receiver gather of the strip configuration for the central-receiver position; (a) including the water surface; (b) without the water surface; (c) results of the removal procedure.

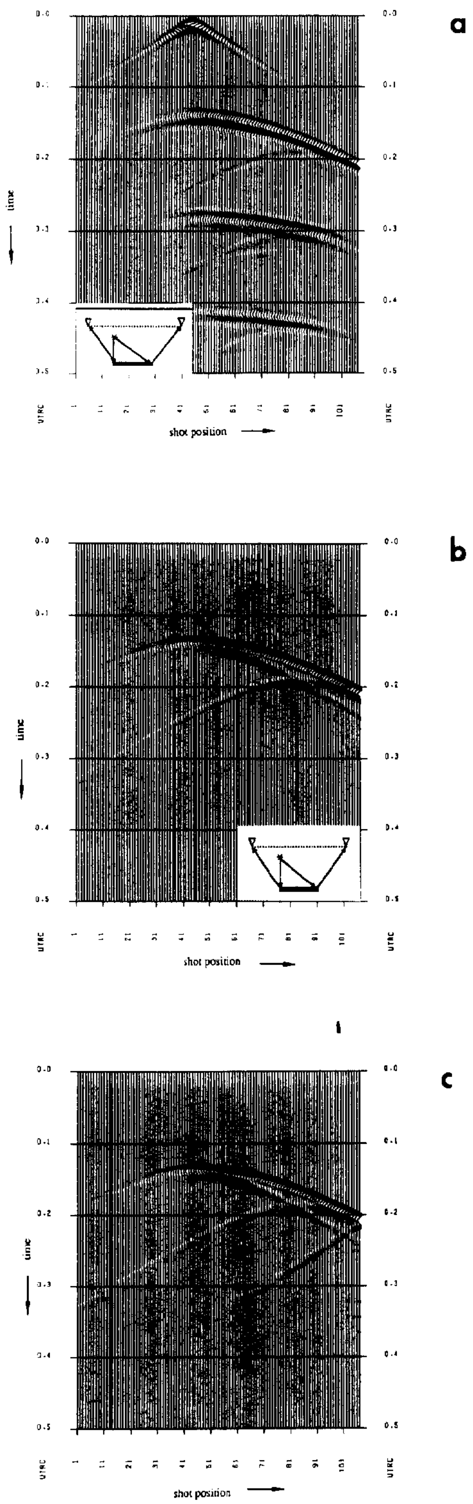

Fig. 4. Common-receiver gather of the strip configuration for the receiver position above the left-hand edge; (a) including the water surface; (b) without the water surface; (c) results of the removal procedure.