# Probing the solution space of an EM inversion problem with a genetic algorithm

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## SUMMARY

In an inversion for the subsurface conductivity distribution using frequency-domain Controlled-Source Electromagnetic data, various amounts of horizontal components may be included. We investigate which combination of components are best suited to invert for a vertical transverse isotropic (VTI) subsurface. We do this by probing the solutionspace using a genetic algorithm. We found, by studying a simple horizontally layered medium, that if only electric data are used, either the horizontal or the vertical conductivity of a layer can be estimated properly, but not both. Including the crossline electric field does not add additional information. In contrast, including the two horizontal magnetic components along with the two horizontal electric components allows to retrieve a better estimate of some of the VTI parameters. For an isotropic subsurface, the electric field is sufficient to invert for the subsurface conductivity.


## INTRODUCTION

In most processing flows of Controlled-Source Electromagnetic (CSEM) data, an inversion for the subsurface conductivity distribution is carried out. There are many ways to do this inversion and, correspondingly, many publications about the topic: 1D (e.g., Christensen and Dodds, 2007; Key, 2009) or 3D (e.g., Grayver et al., 2014), constrained by seismic data (e.g., Brown et al., 2012), combined with magnetotelluric (MT) data (e.g., Sasaki, 2013; Wiik et al., 2013) or using CSEM data only (e.g., Ray et al., 2013) just to name a few.

In this study, we are dealing with a synthetic 3D monochromatic frequency-domain CSEM survey above a horizontally layered vertical transverse isotropic (VTI) subsurface. The latter means, that each layer is characterized by a vertical conductivity $\sigma^V$ and a horizontal conductivity $\sigma^H$. We aim to investigate which horizontal components of the electromagnetic field (electric and magnetic) are required to successfully invert for the subsurface conductivity distribution.

One can use only the inline electric field $E_x$ for the inversion (option 1). That is quite fast, because the forward problem is reduced to one component, but possible information recorded by the other three components remains unused. Another option is to use the inline electric field $E_x$ as well as the crossline electric field $E_y$ in a joint inversion (option 2). How much information does the magnetic field add to an inversion problem? This question suggests to invert all four horizontal components $E_x$, $E_y$, $H_x$ and $H_y$ in a joint inversion (option 3), which makes the forward problem computationally intense, but uses all available information. All three inversion options are summarized in Table 1.

Table 1: Overview of the three inversion strategies investigated.

| option | number of components | components |
|---|---|---|
| 1 | 1 | $E_x$ |
| 2 | 2 | $E_x$, $E_y$ |
| 3 | 4 | $E_x$, $E_y$, $H_x$, $H_y$ |

The shape of the solutionspace around the global minimum is important for determining if an algorithm can find the correct solution located at the center of the global minimum. Ideally, the global minimum has a broad cone of attraction and ends in a point at the correct solution. If the cone of attraction is very small, it is unlikely to find the global minimum. A conjugate gradient scheme would need a starting model very close to the correct solution and a search algorithm based on a random search pattern would have a small chance to hit the global minimum. If the global minimum is very broad at the bottom, the correct solution will not be found even though the inversion algorithm has converged into the global minimum. In that case, any solution in the flat bottom of the global minimum may be picked.

Because the solutionspace is $n$-dimensional, where $n$ is the amount of unknowns, plotting the solutionspace for a problem with more than three unknowns is not practical. Therefore, we aim to probe the solutionspace for the previously mentioned three options using a genetic algorithm (e.g., Goldberg, 1989). In contrast to, for example, a conjugate gradient method, which can optimize a problem for thousands of unknown parameters, a genetic algorithm only allows to invert for a few parameters. Its big advantage is, that it can jump out of a local minimum and, therefore, the starting model does not need to be as close to the correct solution as for a conjugate gradient method. Another advantage of a genetic algorithm is, that for several runs with the exact same input parameters it will always end up at a slightly different result. Ideally all results of separate runs are situated in the global minimum. It is this last property, which makes the algorithm so interesting for our purpose, because by running the code several times for the same parameters, the global minimum can be probed. A narrow global minimum will not be found by all the runs. A global minimum with a flat bottom in one or more dimensions, meaning a poor sensitivity with respect to that specific parameter, is reflected in a large spread of all the solutions for this parameter.

In the next section we describe the genetic algorithm used for this study in more detail. Then we show results for a very simple experiment with just four unknown parameters. From this experiment, we aim to conclude which of the three options described previously is best suited for an inversion for the subsurface medium parameters of a VTI medium. We are especially interested in investigating if the magnetic field adds complementary information to the problem?


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## METHOD

Our genetic algorithm starts with a population of random individuals of which each represents a possible solution to the problem at hand. For example, if we are inverting for four parameters, one individual consists of these four parameters encoded as a string of characters. The basic idea of genetic algorithms is to evolve these individuals according to evolutionary theory to a solution very close to the correct one. For each individual a misfit function is computed. We use the following least-squares-type misfit function:

$$\sum_{i=1}^{N} \frac{\left[\Re\left(d_i\right)-\Re\left(d_i(m)\right)\right]^2+\left[\Im\left(d_i\right)-\Im\left(d_i(m)\right)\right]^2}{\left[\Re\left(d_i\right)\right]^2+\left[\Im\left(d_i\right)\right]^2+\varepsilon}, \qquad (1)$$

where $\Re$ is the real part, $\Im$ is the imaginary part and $N$ is the amount of samples in the data $d$. The forward modeled data for a set of model parameters $m$ are $d(m)$. To compute the forward modeled data, we use an efficient analytical modeling package for layered VTI-media, which should become available as an open-source resource this year (Hunziker et al., 2014). The regularization parameter $\varepsilon$ is set to $10^{-20}$. The normalization is introduced into the misfit function in order to make the misfit for the three different inversion options comparable.

Individuals with a smaller misfit are considered fitter in the sense of evolutionary theory and, thus, have a larger chance to be selected for the next generation. There is a predefined chance, that an individual, which is selected to go to the next generation, is altered by crossover or mutation. In the process of crossover, the strings encoding the parameters of two individuals are cut at a random location and, subsequently, the tails of the two strings are exchanged. This allows to recombine different parameters. In the process of mutation, one symbol in the string encoding the parameters of one individual is exchanged with a random other symbol allowing to test a new randomly chosen value for one of the encoded parameters. The best solutions are also allowed to pass on to the next generation unaltered (eliting) in order to ensure that a good solution is not lost. Additionally, in each generation a set of random new individuals enter the population (migration). This increases the chance to leave a local minimum.

## RESULTS

For this study, we use a simple model depicted in Figure 1. Although we invert for vertical conductivity $\sigma^V$ as well as for horizontal conductivity $\sigma^H$, the model itself is isotropic ($\sigma^V = \sigma^H$). Choosing the same value for $\sigma^V$ and $\sigma^H$ per layer allows a fairer comparison of the relative sensitivity to each other. We invert for the vertical and horizontal conductivity of the layer below the ocean bottom and of the reservoir layer. These layers are marked with a question mark in Figure 1. Thus, we invert for four parameters. All other parameters, including the thickness of the layers, are fixed. Note that this setup is not intended for practical applications, but for testing different inversion approaches.

We run the genetic algorithm for 150 generations with each generation consisting of 560 individuals. The starting population consists of random values in the range of $10^{-4}$ and $10^{1}$ S/m. The genetic algorithm was run nine times for each option. Figure 2 shows the misfit of the best solution of all nine runs as a function of generations for all three inversion options. If only the electric field is used (options 1 and 2) the best misfit achieved is around 1. This is only the case for two out of nine runs. This is in contrast to option 3 which includes the magnetic field. In that case five out of nine runs have a misfit below 1. This suggests, that the cone of attraction of the global minimum is broadened by adding the magnetic field, such that more solutions end up in the global minimum.

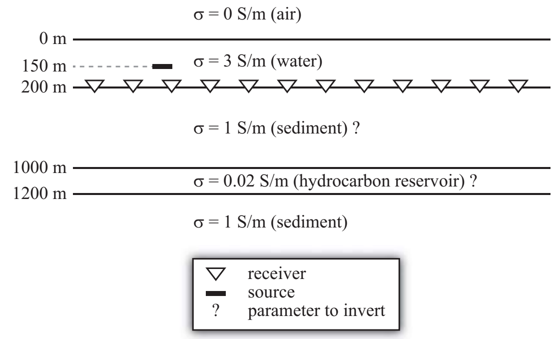


Figure 1: The model used for the simulations. The isotropic conductivity $\sigma$ is given for each layer. Conductivities with a question mark are inverted for.

Figure 3 shows the solutions found by the nine runs (blue asterisk) for the first layer below the ocean bottom on the left and for the second layer (reservoir) below the ocean bottom on the right. The correct solution is indicated by a red circle. Considering Figure 2, it is not surprising that the plots for option 1 ($E_x$ only, Figure 3a and 3b) are quite similar to the plots of option 2 ($E_x$ and $E_y$, Figure 3c and 3d). If only the electric field is used in the inversion, $\sigma^H$ of the first layer and $\sigma^V$ of the reservoir layer are well defined, but the two other parameters not. The global minimum for the other two parameters must be flat and long.

Note, that the electric field is a transverse field. That means that a vertically diffusing signal is sensitive to the horizontal medium parameter and a horizontally diffusing signal to the vertical medium parameter. This indicates in a very simplistic view, that most of the signal that is recorded at the receivers diffuses almost vertically down to the reservoir leading to the strong sensitivity to the horizontal conductivity of the first layer. At the reservoir, the signal is refracted and travels horizontally along the reservoir resulting in the strong sensitivity to the vertical conductivity of the reservoir. Finally, the signal is emitted vertically back to the receivers contributing again to the strong sensitivity to the horizontal conductivity of the first layer.

Adding the two magnetic field components to the inversion problem, thus inverting $E_x$, $E_y$, $H_x$ and $H_y$ jointly (option 3, Figure 3e and 3f), significantly improves the estimate of $\sigma^V$ of the first layer below the ocean bottom. Also the spread of


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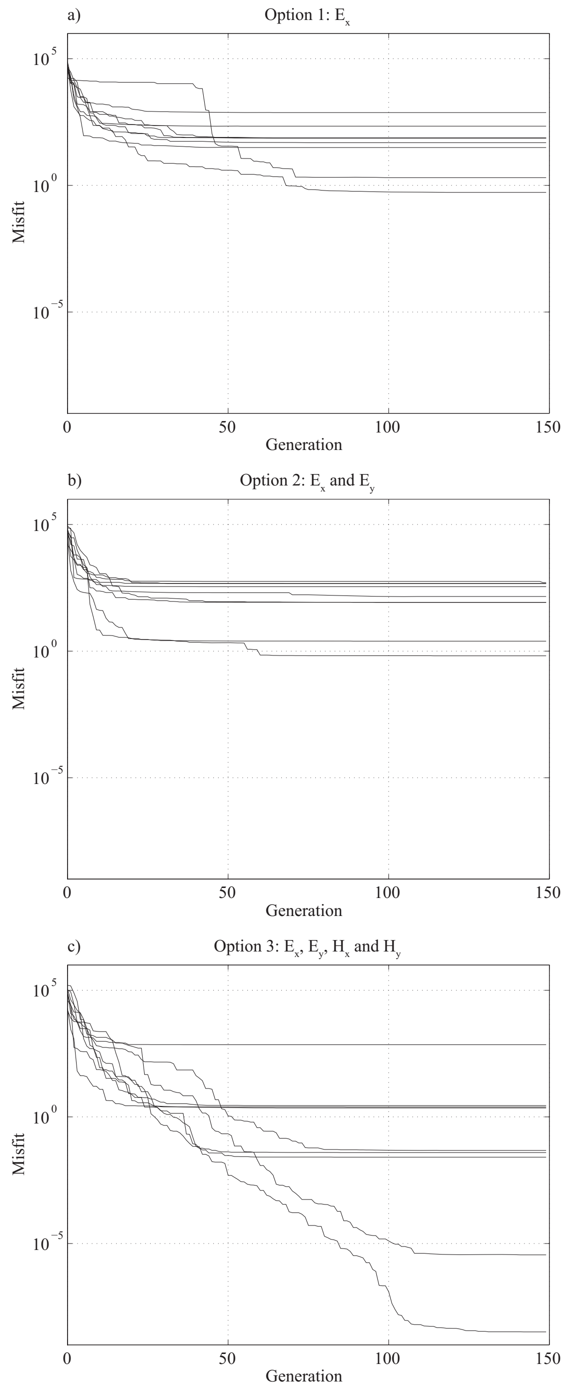


Figure 2: Convergence of the genetic algorithm for the different options as indicated in the header of the plots. For each run, the misfit of the best solution is plotted as a function of generation. Note that the vertical axis is identical for all three plots.

the estimate of $\sigma^H$ of the reservoir layer is slightly reduced, but the effect is much less compared to the improvement of the estimates of the material parameters of the first layer. This indicates again, that the magnetic field adds crucial information to the inversion problem at hand.

## DISCUSSION

This analysis was carried out on a very simple inversion problem. A more complicated medium, which includes reflectors below the reservoir, might help estimating the medium parameters of the reservoir. For a future study, we would also like to include layer thicknesses, noise and measurement errors.

At short offsets, CSEM data are dominated by the direct field and at large offsets, especially in shallow water, by the airwave (Amundsen et al., 2006). Suppressing these events alters the solutionspace of an inversion problem and, thus, might make it easier to find the subsurface conductivity distribution. The benefit would be that fitting of events of large amplitude but without any information about the subsurface can be avoided. Interferometry allows to suppress these events (Hunziker et al., 2013). Therefore, we plan to adapt our inversion code, such that the output of interferometry can be used as well to invert for the subsurface conductivity distribution, allowing to probe the corresponding solutionspace.

Another option would be to invert decomposed fields instead of full fields. Inverting only the upward decaying field also suppresses the direct field and dramatically decreases the effects of the airwave (Amundsen et al., 2006).

## CONCLUSIONS

An inversion that uses only electric data can find either the horizontal or the vertical conductivity of a layer, but not both in the medium tested. Adding the crossline electric field to the inline electric field does not improve the inversion results. However, magnetic data adds complementary information to an EM inversion problem that inverts for the conductivity of a VTI subsurface. This additional information alters the shape of the solutionspace around the global minimum, firstly, increasing the chance to end up close to the correct solution and, secondly, allowing to retrieve more VTI parameters. In case the medium under investigation is isotropic, the electric field is sufficient to find the conductivity distribution.

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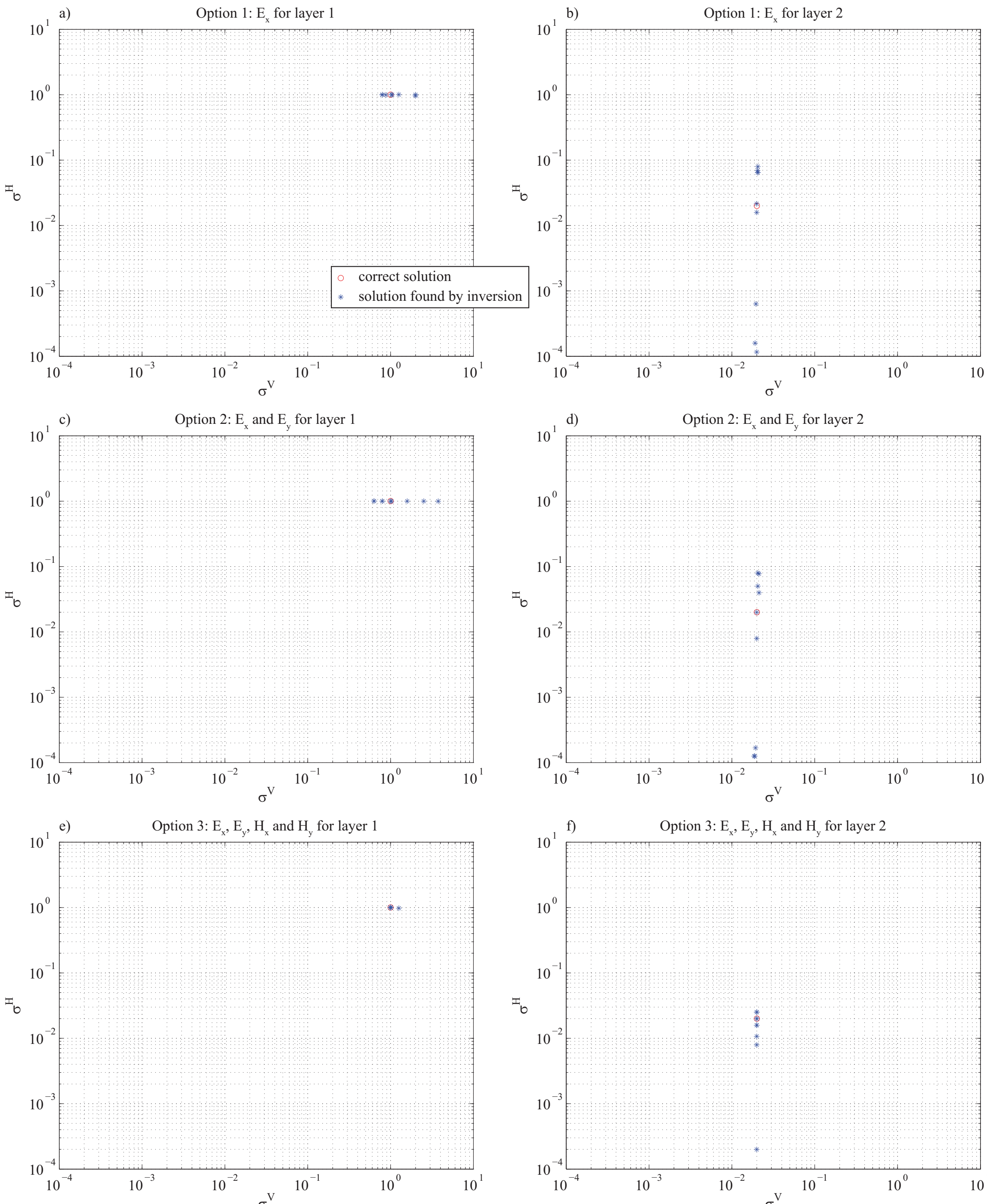


Figure 3: Results of the genetic algorithm for the three different options as indicated in the header of the plots. The parameters of the first layer below the ocean bottom are shown on the left side and the parameters of the reservoir (second layer below the ocean bottom) are shown on the right side.


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**EDITED REFERENCES**

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