DELPHI

9

Areal shot record technology for multi-component data

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9.1 Introduction

In Chapter 8 the method of controlled illumination was discussed for single component data. The aim of this chapter is to generalize this method for the situation of multi-component data. Having multi-component data available means that full elastic processing is feasible. Hence, by properly designing the synthesis operator one can arrive at areal shot records related to controlled P-wave illumination or controlled S-wave illumination. As in the single component case, the control can be put at the surface or in the subsurface. For the latter situation it is useful to distinguish between controlled illumination just below the weathered layer (the 'static problem') or at the target.

In the following sections we will show step by step how the synthesis operator is integrated with the decomposition operator (control at the surface) and with the inverse extrapolation operator (control in the subsurface).

9.2 Forward model of multi-component data

In this section we briefly review the forward model for multi-component seismic data. In the following sections this model will be used as the starting point for the derivation of the synthesis operators. To keep the notation simple we restrict ourselves to the 2-D situation and 2x2 component data (see Wapenaar and Berkhout, 1989, for a more detailed discussion).

At the free surface z_0 a multi-component single-shot record $\vec{V}(z_0)$ can be written as

$$\vec{V}(z_0) = \mathbf{Y}(z_0)\,\vec{T}(z_0), \tag{9.1a}$$

where

$$\vec{V}(z_0) = \begin{pmatrix} \vec{V}_x(z_0) \\ \vec{V}_z(z_0) \end{pmatrix}, \tag{9.1b}$$

$$\mathbf{Y}(z_0) = \begin{pmatrix} \mathbf{Y}_{x,x}(z_0) & \mathbf{Y}_{x,z}(z_0) \\ \mathbf{Y}_{z,x}(z_0) & \mathbf{Y}_{z,z}(z_0) \end{pmatrix} \tag{9.1c}$$

and

$$\vec{T}(z_0) = \begin{pmatrix} \vec{T}_{xz}(z_0) \\ \vec{T}_{zz}(z_0) \end{pmatrix}. \tag{9.1d}$$

In equation (9.1b), $\vec{V}_x(z_0)$ and $\vec{V}_z(z_0)$ are vectors containing the measured particle velocity components $V_x\,(x,z_0,\omega)$ and $V_z\,(x,z_0,\omega)$ discretized along the x-axis, for one frequency component ω. In equation (9.1d), $\vec{T}_{xz}(z_0)$ and $\vec{T}_{zz}(z_0)$ are vectors containing the discretized source stress components $\tau_{xz}(x,z_0,\omega)$ and $\tau_{zz}(x,z_0,\omega)$. Matrix $\mathbf{Y}(z_0)$ is the admittance transfer matrix, relating the measured velocities to the induced stresses.

For a multi-component, *multi*-shot record we write

$$\mathbf{V}(z_0) = \mathbf{Y}(z_0)\,\mathbf{T}(z_0), \tag{9.2a}$$

where

$$\mathbf{V}(z_0) = \begin{pmatrix} \mathbf{V}_{x,x}(z_0) & \mathbf{V}_{x,z}(z_0) \\ \mathbf{V}_{z,x}(z_0) & \mathbf{V}_{z,z}(z_0) \end{pmatrix} \tag{9.2b}$$

and

$$\mathbf{T}(z_0) = \begin{pmatrix} \mathbf{T}_{xz}(z_0) & \mathbf{O} \\ \mathbf{O} & \mathbf{T}_{zz}(z_0) \end{pmatrix}. \tag{9.2c}$$

The columns of the data matrix $\mathbf{V}(z_0)$ contain the data vectors $\vec{V}(z_0)$. Note that the source matrix $\mathbf{T}(z_0)$ must be organized such that the left part only contains shear stress source vectors and the right part only contains tensile stress source vectors.

For a regular grid of point sources the source matrix can be further simplified to

$$\mathbf{T}(z_0) = \mathbf{I}\,S(\omega), \tag{9.3}$$

where **I** is the identity matrix and S(ω) denotes the source function for frequency component ω. The admittance transfer matrix may be written as follows

$$\mathbf{Y}(z_0) = \mathbf{D}^-(z_0)\,\mathbf{X}(z_0,z_0)\,\mathbf{D}^+(z_0), \tag{9.4a}$$

where the one-way transfer matrix **X** is defined as

$$\mathbf{X}(z_0,z_0) = \begin{pmatrix} \mathbf{X}_{\varphi,\varphi} & \mathbf{X}_{\varphi,\psi} \\ \mathbf{X}_{\psi,\varphi} & \mathbf{X}_{\psi,\psi} \end{pmatrix}(z_0,z_0). \tag{9.4b}$$

The first subscript refers to the type of upgoing waves at the receiver side (φ stands for P-waves, ψ stands for S-waves); the second subscript refers to the type of downgoing waves at the source side. The matrix $\mathbf{D}^+(z_0)$ is a decomposition operator, relating the downgoing P- and S-waves to the induced stresses at the free surface z_0; matrix $\mathbf{D}^-(z_0)$ is a composition matrix, relating the particle velocities at the free surface z_0 to the upgoing P- and S-waves (see Delphi Volume I, Appendix II).Note that the direct waves and the surface waves are deleted in equation (9.4a). For the one-way transfer matrix **X** we may write

$$\mathbf{X}(z_0,z_0) = [\mathbf{I} - \mathbf{X}_0(z_0,z_0)\,\mathbf{R}^-(z_0)]^{-1}\,\mathbf{X}_0(z_0,z_0). \tag{9.5}$$

Here $\mathbf{X}_0$ is subdivided in 4 submatrices, in a similar way as **X** in equation (9.4b).

Whereas **X** includes surface related multiples, $\mathbf{X}_0$ describes the relation between downgoing and upgoing waves at z_0, without the surface related multiples. In our process of surface-related multiple elimination $\mathbf{X}(z_0,z_0)$ is transformed to $\mathbf{X}_0(z_0,z_0)$, see Chapter 5. $\mathbf{R}^-$ describes the reflectivity of the free surface for upgoing waves.

Matrix $\mathbf{X}_0(z_0,z_0)$ may be related to the reflection properties of the subsurface according to

$$\mathbf{X}_0(z_0,z_0) = \sum_{n=1}^{M} \mathbf{W}^-(z_0,z_n)\,\mathbf{R}^+(z_n)\,\mathbf{W}^+(z_n,z_0), \tag{9.6a}$$

where

$$\mathbf{W}^+(z_n,z_0) = \begin{pmatrix} \mathbf{W}^+_{\varphi,\varphi} & \mathbf{W}^+_{\varphi,\psi} \\ \mathbf{W}^+_{\psi,\varphi} & \mathbf{W}^+_{\psi,\psi} \end{pmatrix}(z_n,z_0) \tag{9.6b}$$

$$\mathbf{W}^-(z_0,z_n) = \begin{pmatrix} \mathbf{W}^-_{\varphi,\varphi} & \mathbf{W}^-_{\varphi,\psi} \\ \mathbf{W}^-_{\psi,\varphi} & \mathbf{W}^-_{\psi,\psi} \end{pmatrix}(z_0,z_n) \tag{9.6c}$$

and

$$\mathbf{R}^+(z_n) = \begin{pmatrix} \mathbf{R}^+_{\varphi,\varphi} & \mathbf{R}^+_{\varphi,\psi} \\ \mathbf{R}^+_{\psi,\varphi} & \mathbf{R}^+_{\psi,\psi} \end{pmatrix}(z_n). \tag{9.6d}$$

Here $\mathbf{W}^+$ and $\mathbf{W}^-$ are wave field extrapolation operators for downgoing (+) and upgoing (–) waves, respectively and $\mathbf{R}^+$ describes the reflectivity in the subsurface at depth level z_n for downgoing waves.

In analogy with equation (9.6a) we define a one-way transfer matrix $\mathbf{X}_0(z_m,z_m)$ at depth level z_m according to

$$\mathbf{X}_0(z_m,z_m) = \sum_{n=m}^{M} \mathbf{W}^-(z_m,z_n)\, \mathbf{R}^+(z_n)\, \mathbf{W}^+(z_n,z_m). \tag{9.7}$$

Ignoring the 'overburden response' (i.e., the response from the medium between the surface z_0 and depth level z_m), we may relate $\mathbf{X}_0(z_0,z_0)$ to $\mathbf{X}_0(z_m,z_m)$, according to

$$\mathbf{X}_0(z_0,z_0) = \mathbf{W}^-(z_0,z_m)\, \mathbf{X}_0(z_m,z_m)\, \mathbf{W}^+(z_m,z_0), \tag{9.8}$$

where we used the properties

$$\mathbf{W}^+(z_n,z_0) = \mathbf{W}^+(z_n,z_m)\, \mathbf{W}^+(z_m,z_0) \tag{9.9a}$$

and

$$\mathbf{W}^-(z_0,z_n) = \mathbf{W}^-(z_0,z_m)\, \mathbf{W}^-(z_m,z_n). \tag{9.9b}$$

The total forward model, described by equations (9.2a), (9.3), (9.4a), (9.5) and (9.8) is visualized in Fig. 9.1.

9.3 Controlled illumination at the surface, theory

Consider the combination of equations (9.2a), (9.3) and (9.4a):

$$\mathbf{V}(z_0) = \mathbf{D}^-(z_0)\, \mathbf{X}(z_0,z_0)\, \mathbf{D}^+(z_0)\, S(\omega), \tag{9.10}$$

which is again the forward model for a multi-component multi-shot record at the free surface. Controlled illumination at the surface should result in a (synthesized) multi-component *areal* shot record given by the following forward model:

$$\vec{V}_{syn}(z_0) = \mathbf{D}^-(z_0)\, \mathbf{X}(z_0,z_0)\, \vec{\Gamma}^+(z_0)\, S(\omega), \tag{9.11a}$$

where

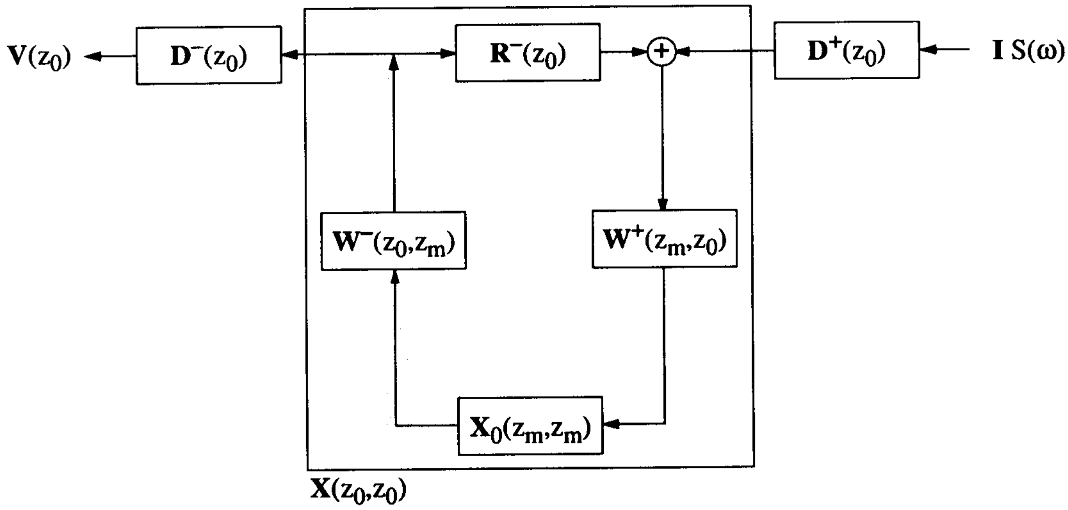

Fig. 9.1 *Forward model of multi-component seismic data. Note that the direct waves and the response of the overburden (between z_0 and z_m) are deleted.*

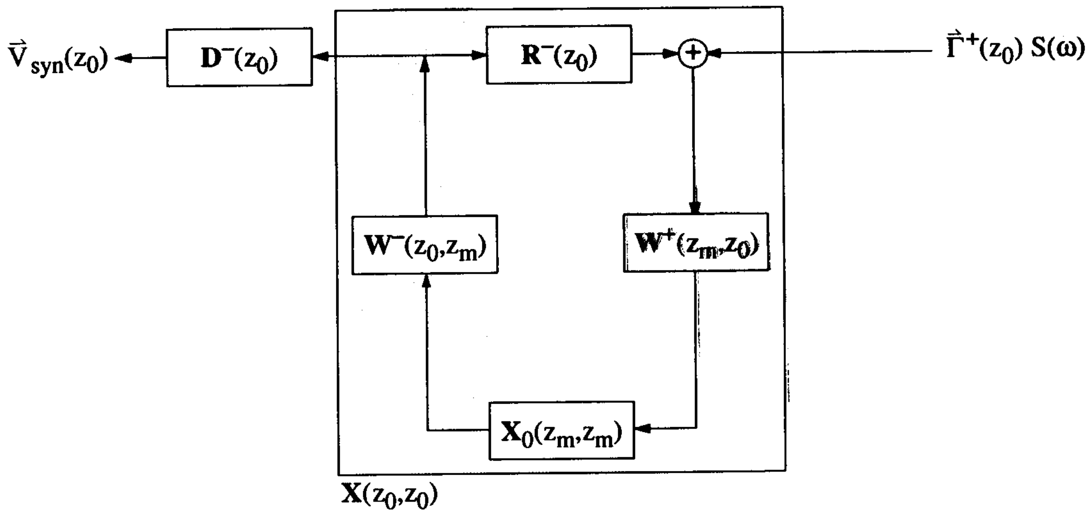

Fig. 9.2 *Forward model of a multi-component shot record with controlled illumination at the surface. The vector $\vec{\Gamma}^+$ contains any desired combination of downgoing P- and /or S-waves at z_0.*

$$\vec{\Gamma}^+(z_0) = \begin{pmatrix} \vec{\Gamma}^+_\varphi(z_0) \\ \vec{\Gamma}^+_\psi(z_0) \end{pmatrix}, \tag{9.11b}$$

see Fig. 9.2.

Here $\vec{\Gamma}^+_\varphi$ and $\vec{\Gamma}^+_\psi$ are vectors containing the (scaled) desired downgoing P- and S-wave fields at the surface. For instance, when a horizontal downgoing plane P-wave is required, then $\vec{\Gamma}^+_\psi$ equals zero, whereas $\vec{\Gamma}^+_\varphi$ reads

$$\vec{\Gamma}^+_\varphi(z_0) = [1,1,...,1]^T. \tag{9.12}$$

The desired downgoing source wave field $\vec{\Gamma}^+(z_0)\ S(\omega)$ can be related to a (synthetic) source stress vector $\vec{T}_{syn}(z_0)$ via the decomposition operator $\mathbf{D}^+(z_0)$, according to

$$\vec{\Gamma}^+(z_0)\ S(\omega) = \mathbf{D}^+(z_0)\ \vec{T}_{syn}(z_0). \tag{9.13}$$

Note that vector $\vec{T}_{syn}(z_0)$ contains the stress distribution that should be applied at the surface in order to generate the desired downgoing wave field $\vec{\Gamma}^+(z_0)$.

Upon substitution of this expression in equation (9.11a) we find

$$\vec{V}_{syn}(z_0) = \mathbf{D}^-(z_0)\ \mathbf{X}(z_0,z_0)\ \mathbf{D}^+(z_0)\ \vec{T}_{syn}(z_0). \tag{9.14}$$

Using equation (9.10) we now easily find that the synthesis process is formulated by the following equation:

$$\vec{V}_{syn}(z_0) = \mathbf{V}(z_0)\ \vec{T}'_{syn}(z_0), \tag{9.15a}$$

where the dimensionless synthesis operator is given by

$$\vec{T}'_{syn}(z_0) = S^{-1}(\omega)\ \vec{T}_{syn}(z_0), \tag{9.15b}$$

or, using equation (9.13)

$$\vec{T}'_{syn}(z_0) = [\mathbf{D}^+(z_0)]^{-1}\ \vec{\Gamma}^+(z_0). \tag{9.15c}$$

Just as in the single-component situation, the synthesis process described by equation (9.15a) involves a weighted addition of the shot records contained in the columns of the data matrix $\mathbf{V}(z_0)$. Equation (9.15c) shows that the decomposition process is integrated in the synthesis process.

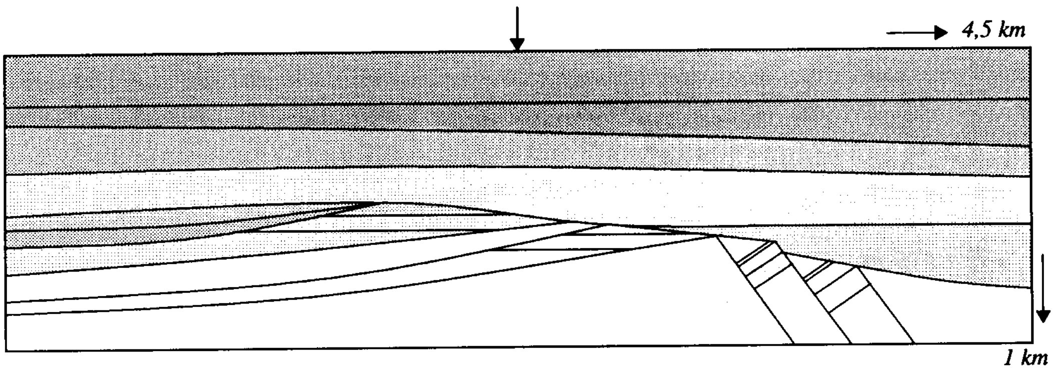

Fig. 9.3 The Brent model.

9.4 Controlled illumination at the surface, example

This straightforward synthesis process is now illustrated with an example. We have used the synthetic 'Brent' data set which is described in Delphi Volume II, Appendix I. The model is shown in Fig. 9.3.

The 2x2 component 2-D data set consists of 256 shot records and 512 fixed receiver positions on top of the model. This dataset was generated by using a staggered grid finite difference program. One 2x2 component shot record is shown, in the space time domain, in Fig. 9.4. Note that we have muted the direct field and removed the Rayleigh wave by velocity filtering in the wavenumber-frequency domain.

Applying the synthesis process as described in equation (9.15a) for a horizontal downgoing plane P-wave (see equation (9.12)) yields the results shown in Fig. 9.5. Applying a receiver decomposition[1] for upgoing P-waves as well yields the P-P response in Fig. 9.6. Following a similar procedure for downgoing S-wave synthesis and upgoing S-wave decomposition yields the results shown in Fig. 9.7 and Fig. 9.8, respectively. Note that at the left and right bottom of Fig. 9.8 some data is missing due to the removal of the boundary effects of the finite difference modelling. The P-P and S-S plane wave responses (Fig. 9.6 and Fig. 9.8) give already a good indication of the subsurface model (Fig. 9.3). It is also observed that the S-S data has a better resolution due to the smaller wavelength.

1. The receiver decomposition was carried out by applying an optimized decomposition operator of 29 points as derived in Appendix D.

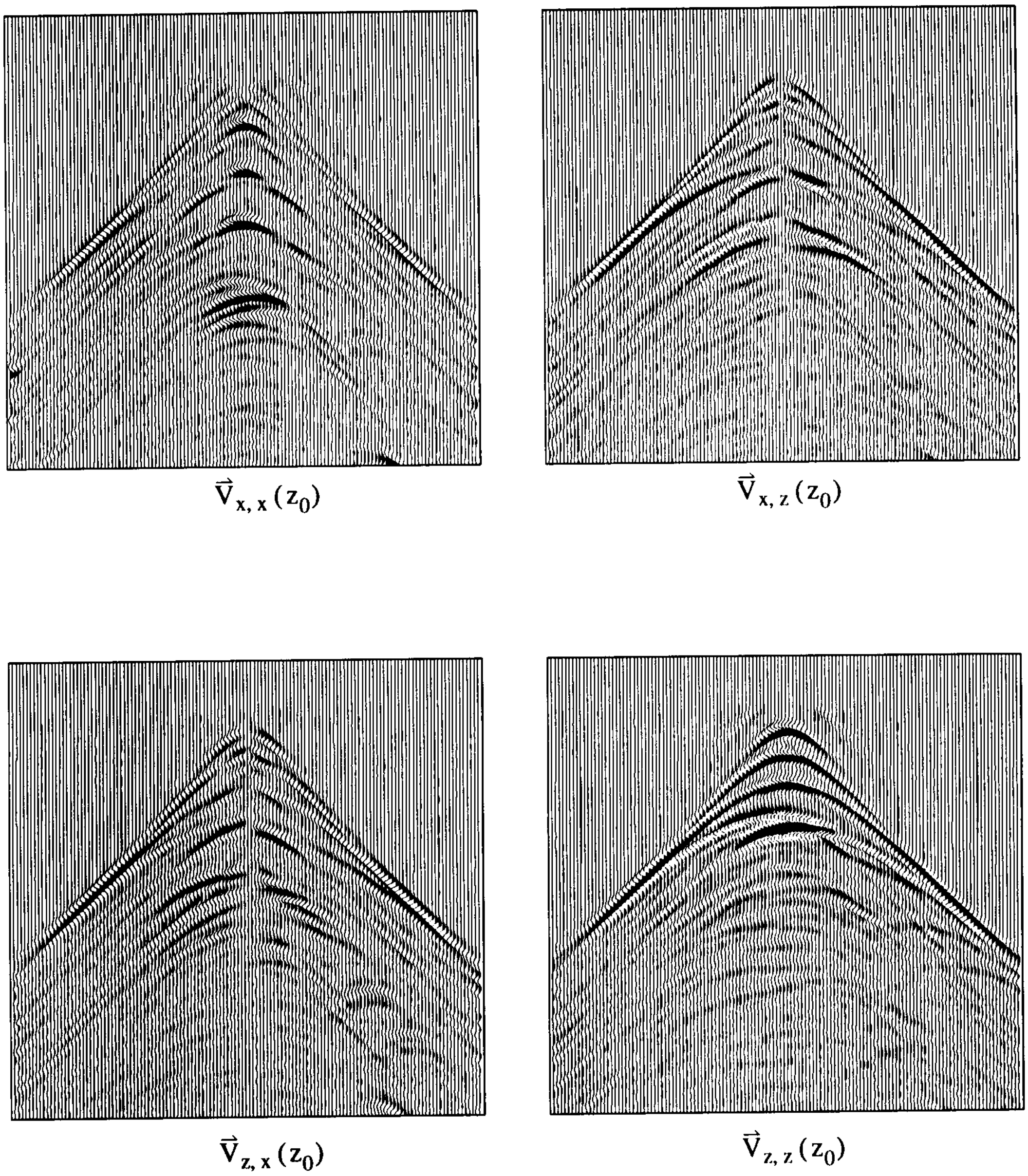

Fig. 9.4 *One 2x2 component shot record for the Brent model. The shot position is indicated by the arrow in Fig. 9.3.*

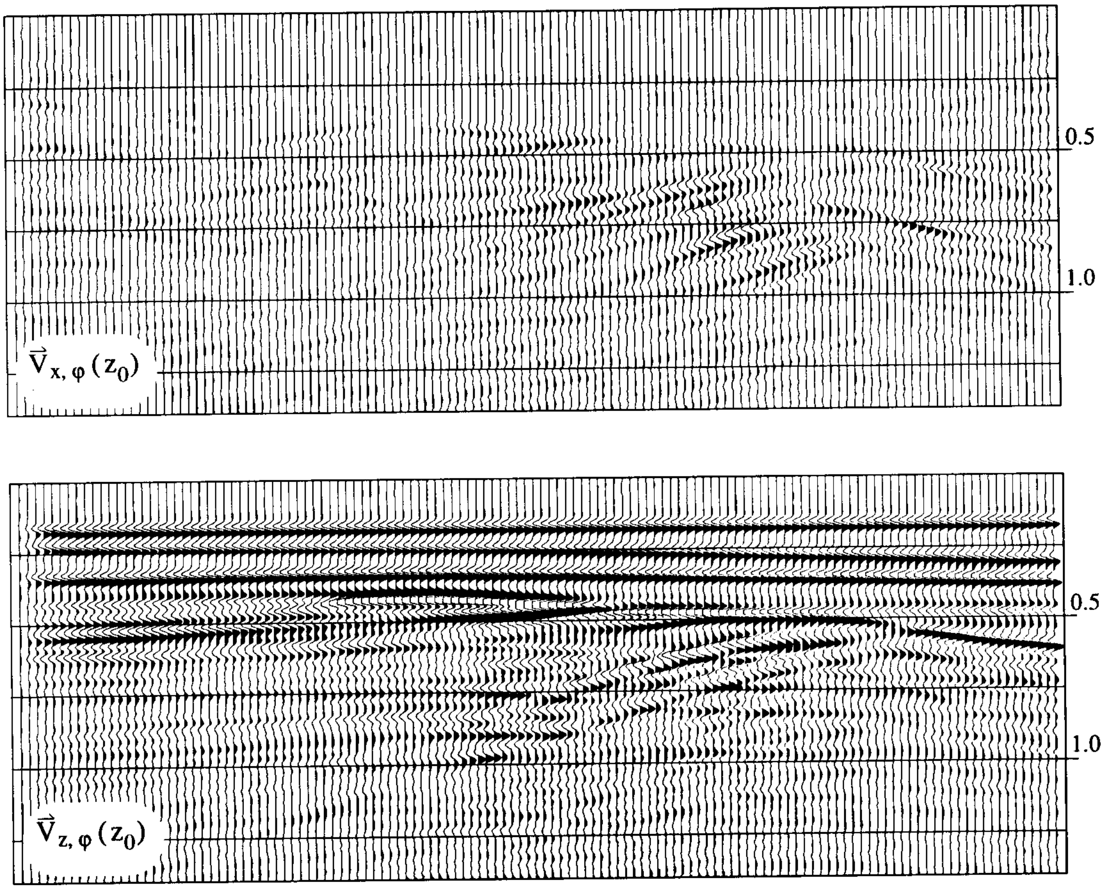

Fig. 9.5 *Synthesized P-wave record for the Brent model.*

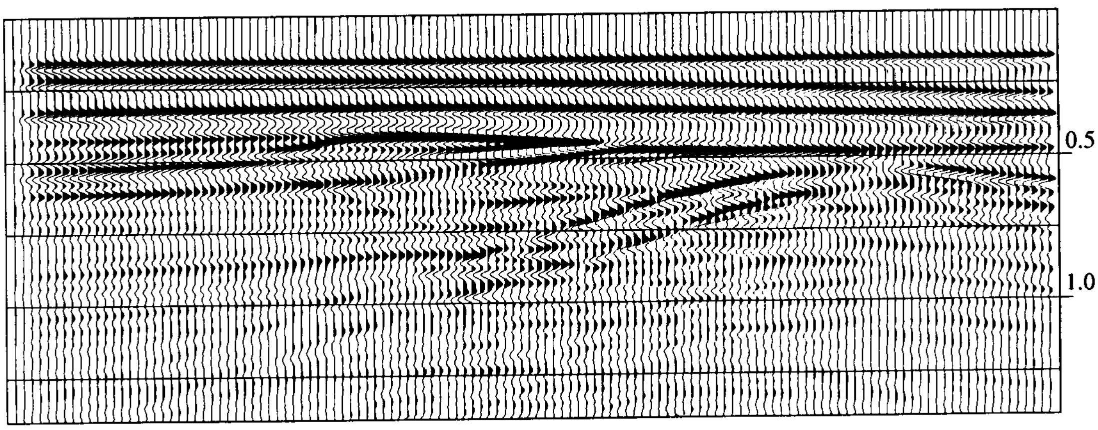

Fig. 9.6 *P-P plane wave response, obtained by applying receiver decomposition to the data of Fig. 9.5.*

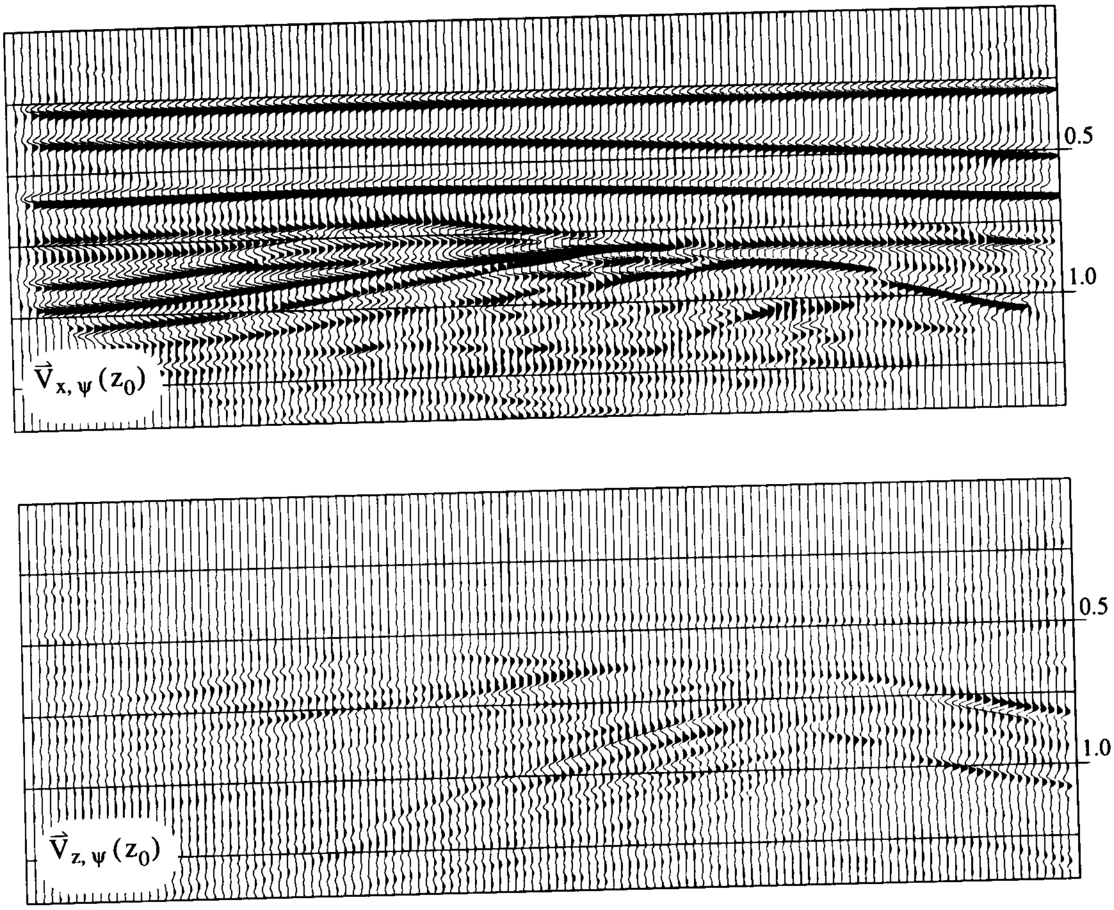

Fig. 9.7 *Synthesized S-wave record for the Brent model.*

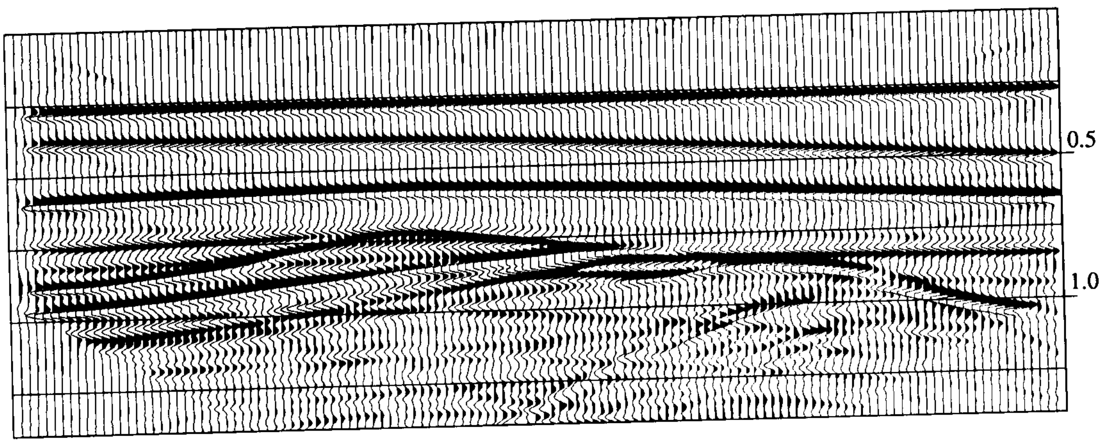

Fig. 9.8 *S-S plane wave response, obtained by applying receiver decomposition to the data of Fig. 9.7.*

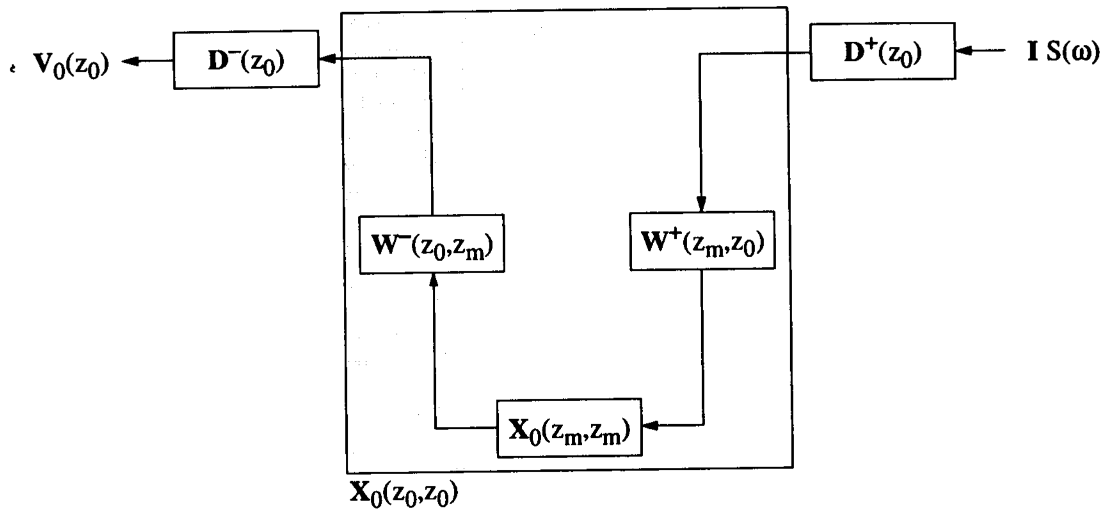

Fig. 9.9 Primary forward model of multi-component seismic data

9.5 Controlled illumination in the subsurface

To keep the discussion simple, we only consider the primary response. Hence, we replace equation (9.10) by

$$\mathbf{V}_0(z_0) = \mathbf{D}^-(z_0)\, \mathbf{X}_0(z_0,z_0)\, \mathbf{D}^+(z_0)\, S(\omega), \tag{9.16a}$$

or, using equation (9.8),

$$\mathbf{V}_0(z_0) = \mathbf{D}^-(z_0)\, \mathbf{W}^-(z_0,z_m)\, \mathbf{X}_0(z_m,z_m)\, \mathbf{W}^+(z_m,z_0)\, \mathbf{D}^+(z_0)\, S(\omega), \tag{9.16b}$$

see Fig. 9.9.

Note that z_m may denote any depth level in the subsurface, for instance the bottom of the weathering layer of the top of the target zone. Controlled illumination at depth level z_m should result in a (synthesized) multi-component areal shot record given by the following forward model:

$$\vec{V}_{0,syn}(z_0) = \mathbf{D}^-(z_0)\, \mathbf{W}^-(z_0,z_m)\, \mathbf{X}_0(z_m,z_m)\, \vec{\Gamma}^+(z_m)\, S(\omega), \tag{9.17}$$

with $\vec{\Gamma}^+(z_m)$ defined in a similar way as $\vec{\Gamma}^+(z_0)$ in equation (9.11b), see Fig. 9.10.

The desired downgoing source wave field $\vec{\Gamma}^+(z_m)\, S(\omega)$ can be related to a (synthetic) source stress vector $\vec{T}_{syn}(z_0)$ via the decomposition operator $\mathbf{D}^+(z_0)$ and the wave field extrapolation operator $\mathbf{W}^+(z_m,z_0)$, according to

$$\vec{\Gamma}^+(z_m)\, S(\omega) = \mathbf{W}^+(z_m,z_0)\, \mathbf{D}^+(z_0)\, \vec{T}_{syn}(z_0). \tag{9.18}$$

Note that vector $\vec{T}_{syn}(z_0)$ contains the stress distribution that should be applied at the surface in order to generate the downgoing wave field $\vec{\Gamma}^+(z_m)$. Upon substitution of this expression in equation (9.17) we find

$$\vec{V}_{0,syn}(z_0) = \mathbf{D}^-(z_0)\, \mathbf{W}^-(z_0,z_m)\, \mathbf{X}_0(z_m,z_m)\, \mathbf{W}^+(z_m,z_0)\, \mathbf{D}^+(z_0)\, \vec{T}_{syn}(z_0). \tag{9.19}$$

Using equation (9.16b) we now easily find that the synthesis process is formulated by the following equation

$$\vec{V}_{0,syn}(z_0) = \mathbf{V}_0(z_0)\, \vec{T}'_{syn}(z_0), \tag{9.20a}$$

where the dimensionless synthesis operator is given by

$$\vec{T}'_{syn}(z_0) = S^{-1}(\omega)\, \vec{T}_{syn}(z_0), \tag{9.20b}$$

or, using equation (9.18),

$$\vec{T}'_{syn}(z_0) = [\mathbf{D}^+(z_0)]^{-1}\, [\mathbf{W}^+(z_m,z_0)]^{-1}\, \vec{\Gamma}^+(z_m), \tag{9.20c}$$

or, using the match filter approach,

$$\vec{T}'_{syn}(z_0) = [\mathbf{D}^+(z_0)]^{-1}\, [\mathbf{W}^-(z_0,z_m)]^*\, \vec{\Gamma}^+(z_m), \tag{9.20d}$$

where * denotes complex conjugation. Again the synthesis process is described as a weighted addition of shot records (equation (9.20a)).Equation (9.20d) shows that the decomposition process and the inverse extrapolation process are integrated with the synthesis process. Note that

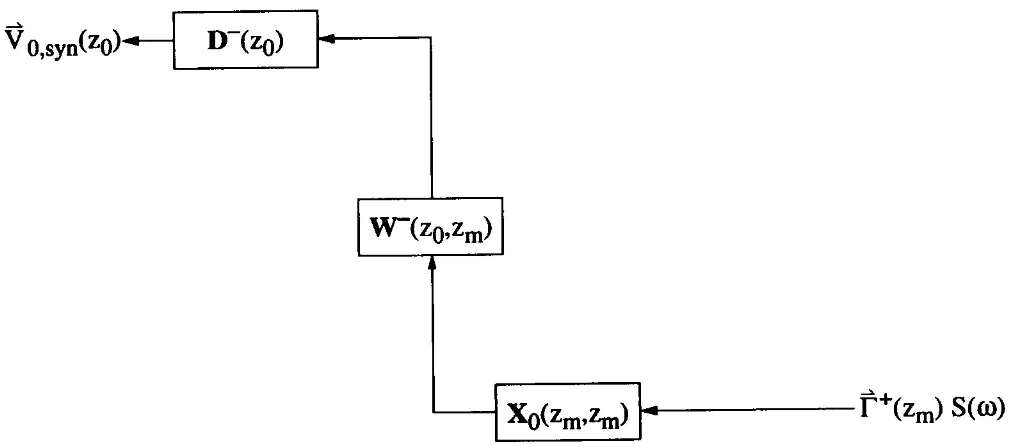

Fig. 9.10 *Primary forward model of a multi-component areal shot record with controlled illumination in the subsurface.*

when fine layering effects between z_0 and z_m play an important role, then the matched filter operator in equation (9.20d) should be replaced by the modified matched filter operator defined by equation (2.12) in Chapter 2.

9.6 Conclusions

It has been shown that the extension of the controlled illumination process to the multicomponent situation leads to controlled P-wave and/or controlled S-wave illumination.

For the single-component situation the synthesis process has always been carried out after preprocessing (decomposition, multiple elimination, Rietveld et al, 1992). For the multi-component situation we have integrated the decomposition process with the synthesis process. Therefore, for controlled illumination at the surface as well as in the subsurface the expressions for the synthesis operator contain the decomposition operator (see equations (9.15c) and (9.20d)).

Throughout this chapter, we have assumed that multi-component sources as well as multi-component receivers are available. In practice multi-component data often means: multi-component receivers and single-component sources. For instance, for vertical stress sources, matrix **V**, as defined in equation (9.2b), should be replaced by

$$\mathbf{V}(z_0) = \begin{pmatrix} \mathbf{V}_{x,z}(z_0) \\ \mathbf{V}_{z,z}(z_0) \end{pmatrix}. \tag{9.21}$$

In this case the synthesis processes as given by equations (9.15a) and (9.20a) should be applied to the transposed of this matrix:

$$\mathbf{V}^T(z_0) = \left(\mathbf{V}^T_{x,z}(z_0) \;\; \mathbf{V}^T_{z,z}(z_0) \right), \tag{9.22a}$$

or, using reciprocity

$$\mathbf{V}^T(z_0) = \left(\mathbf{V}_{z,x}(z_0) \;\; \mathbf{V}_{z,z}(z_0) \right). \tag{9.22b}$$

Note that the latter matrix may be interpreted as single component receiver data (z-component) related to multi-component sources.

Hence, after synthesis, the data are interpreted as single-component receiver data, related to controlled P-wave or controlled S-wave illumination at z_0 or z_m.

9.7 References

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Rietveld, W.E.A., Berkhout, A.J., and Wapenaar, C.P.A., 1992, *Optimum seismic illumination of hydrocarbon reservoirs*: Geophysics, **57**, 1334-1345.

Wapenaar, C.P.A., and Berkhout, A.J., 1989, *Elastic wavefield extrapolation: Redatuming of single- and multicomponent seismic data*: Elsevier Science Publ. Co., Inc.