

SCATTERING BY A STRIP
IN A HOMOGENEOUS MEDIUM

Jan Thorbecke

18 January 1991

Delft University of Technology
Faculty of Mining and Petroleum Engineering
Section Applied Geophysics
Delft
The Netherlands

Title : Scattering by a strip in a homogeneous medium.

Author : Jan Thorbecke

Date : 18 January 1991

Laboratory : Applied Geophysics

Report number : 1991-2

Abstract code : PA 91.90

Address : Delft University of Technology
Dept. of Mining and Petroleum Engineering
Section of Applied Geophysics
P.o. Box 5028
2600 GA Delft
The Netherlands

[122]

Ik ben altijd ontsteld wanneer ik iets voltooi. Ik schrik en wordt door verdriet overmand. Mijn volmaaktheidsinstinct zou me moeten beletten iets te voltooien; het zou me zelfs moeten beletten ergens aan te beginnen. Maar ik ben verstrooid en doe het toch, met als resultaat een produkt dat bij mij niet voortkomt uit de wil, maar uit de afwezigheid ervan. Ik begin omdat ik geen kracht heb om te denken; ik voltooi omdat mijn ziel de kracht mist om eerder op te houden. Dit boek is mijn lafheid.

Fernando Pessoa, *Het Boek Der Rusteloosheid*

Preface

Two years ago, in December 1988, I was ready to start with the last few steps of my study, but I couldn't find any motivation to start with it. I was looking for something I would never find; I was asking questions which I couldn't answer. I considered stopping to study, but I couldn't find any reason why I should stop, neither could I find any reason to finish it. For some reasons, of which I don't care anymore of not knowing them, my last steps of the study came to an end.

This thesis represents the results of my final project at the Department of Mining and Petroleum Engineering, section Applied Geophysics, at the Delft University of Technology. The accompany of this project was done by J.T. Fokkema and P.M. van den Berg, which I would like to thank for their support and understanding. I also would like to thank prof. A.M. Ziolkowski for reading this report and making many helpful comments and suggestions and, everybody who was at the 'Geophysics room', to whom I could ask everything, for helping me with all aspects of my final project and for trying to make me feel at home.

The program I have written, which is based on this report, will be used to produce a reference data set for a new theory about the removal of surface related wave phenomena in the marine case which is developed by J.T. Fokkema and P.M. van den Berg.

Contents

	pagenumber
Abstract	1
Notations and conventions	2
Introduction	5
Chapter 1 Scattering by a strip in an unbounded medium	
1.0 Introduction	6
1.1 Rayleigh's reciprocity theorem	7
1.2 Green's function in a 2-Dimensional space	8
1.2.1 <i>Green's function for the Helmholtz equation</i>	9
1.2.2 <i>The injection source Green's function</i>	12
1.2.3 <i>The force source Green's function</i>	13
1.3 Scattering by a strip	14
1.3.1 <i>Representation for the scattered pressure field</i>	14
1.3.2 <i>Representation for the scattered particle velocity field</i>	18
1.4 Scattering by a perfectly compliant strip	20
1.5 Scattering by a perfectly rigid strip	24
Chapter 2 Scattering by a strip in a 2-dimensional halfspace	
2.0 Introduction	29
2.1 Rayleigh's reciprocity theorem	29
2.1.1 <i>Representation for the pressure field</i>	29
2.1.2 <i>Representation for the particle velocity field</i>	30
2.2 Calculating the Green's function	31
2.3 Scattering in a halfspace	33
2.3.1 <i>Scattering by a perfectly compliant strip</i>	33
2.3.2 <i>Scattering by a perfectly rigid strip</i>	34

Chapter 3 Source and receiver

3.0 Introduction	37
3.1 Receiver	37
3.2 Line source	38
3.2.1 <i>The incident pressure field</i>	38
3.2.2 <i>The incident particle velocity field</i>	39
3.3 An example	40

Chapter 4 Iterative schemes based on minimization of a uniform error criterion

4.0 Introduction	43
4.1 Direct minimization of the error	44
4.2 Recursive minimization of the error	46
4.2.a <i>Computational scheme for an arbitrary operator T</i>	49
4.3 Selfadjoint operator LT	50
4.3.a <i>Computational scheme for a selfadjoint operator LT</i>	52
4.4 Convergence	53
4.4.1 <i>Convergence for the recursive scheme</i>	53
4.4.2 <i>Convergence for the selfadjoint and positive operator LT</i>	57
4.5 Preconditioning ($T = P$)	58

Chapter 5 Numerical implementation

5.0 Introduction	60
5.1 The operator expression LU	60
5.1.1a <i>Perfectly compliant strip in an unbounded medium</i>	60
5.1.1b <i>Perfectly rigid strip in an unbounded medium</i>	62
5.1.2a <i>Perfectly compliant strip in a halfspace</i>	62
5.1.2b <i>Perfectly rigid strip in a halfspace</i>	63
5.2 The preconditioning operator	64
5.1.1a <i>Perfectly compliant strip in an unbounded medium</i>	64
5.1.1b <i>Perfectly rigid strip in an unbounded medium</i>	65
5.1.2a <i>Perfectly compliant strip in a halfspace</i>	65
5.1.2b <i>Perfectly rigid strip in a halfspace</i>	65
5.3 Discrete Fourier transform	66
5.4 Branch points	70

5.5 Causality	72
5.5.1 <i>Complex conjugate and causality</i>	72
5.5.2 <i>The Hilbert transform and causality</i>	73
Chapter 6 Results	
6.0 Introduction	75
6.1 The sea configuration	75
6.2 Reliability of the results	89
6.2.1 <i>Arrival times and point diffractors</i>	89
6.2.2 <i>The numerical Green's function</i>	93
6.2.3 <i>The infinite compliant strip</i>	106
References	111
Appendix A Errors in the DFT	113
Appendix B The input parameters	117
<i>B.1 The input parameters and computation time</i>	117
<i>B.2 Restrictions on the input parameters</i>	118
Appendix C The program	121