From Reflection to Transmission Data

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Contents and Goal

- Brief introduction Reciprocity Theorems
- From R to T for 1D media
- From R to T for 2D media
- Conclusions

Goal: Using calculated coda in reflection imaging to suppress the effects of internal multiples.

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One-way Reciprocity Theorems

Convolution type:

$$\int_{\partial \mathcal{D}_0} \{P_A^+ P_B^- - P_A^- P_B^+\} d^2\mathbf{x} = \int_{\partial \mathcal{D}_m} \{P_A^+ P_B^- - P_A^- P_B^+\} d^2\mathbf{x}$$

Correlation type:

$$\int_{\partial \mathcal{D}_0} \{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\} d^2\mathbf{x} = \int_{\partial \mathcal{D}_m} \{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\} d^2\mathbf{x}$$

See article *"Relations between reflection and transmission responses of 3-D in-homogeneous media."* by Kees Wapenaar, Jan Thorbecke, Deyan Dragonov 2004, Geoph. J. Int. Vol 156, p. 179-194

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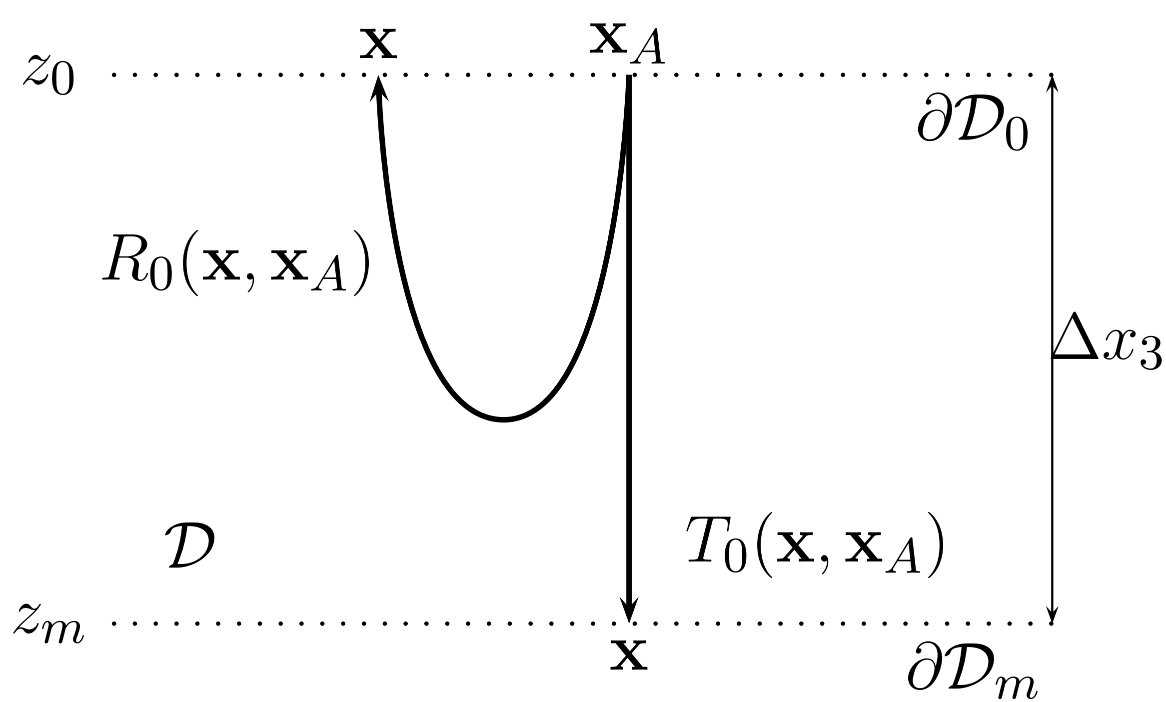
One-way Reciprocity Theorems

Correlation type:

$$\int_{\partial\mathcal{D}_0}\{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\} d^2\mathbf{x} = \int_{\partial\mathcal{D}_m}\{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\} d^2\mathbf{x}$$



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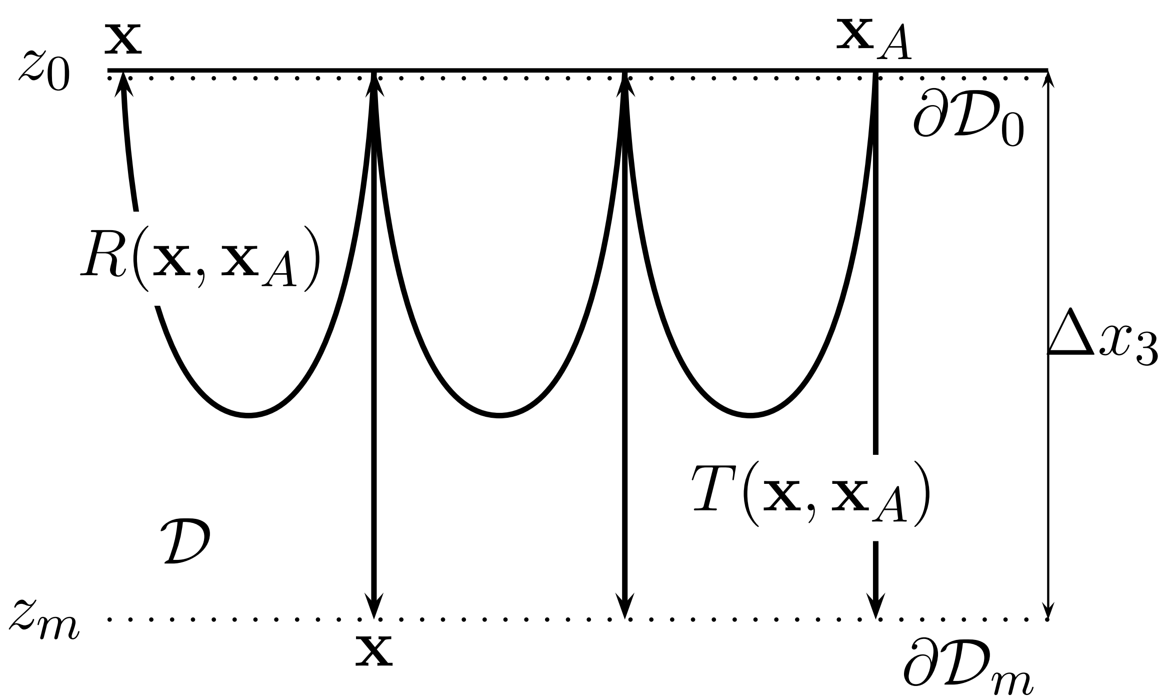
One-way Reciprocity Theorems

Correlation type:

$$\int_{\partial\mathcal{D}_0}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}d^2\mathbf{x} = \int_{\partial\mathcal{D}_m}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}d^2\mathbf{x}$$



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Passive Seismic

$$\int_{\partial\mathcal{D}_0}\{(P_A^+)^*P_B^+-(P_A^-)^*P_B^-\}d^2\mathbf{x}=\int_{\partial\mathcal{D}_m}\{(P_A^+)^*P_B^+-(P_A^-)^*P_B^-\}d^2\mathbf{x}$$

Surface $\partial\mathcal{D}_0$		
Field	**State A**	**State B**
P^+	$\delta(\mathbf{x}_H-\mathbf{x}_{H,A})s_A(\omega)+rP^-$	$\delta(\mathbf{x}_H-\mathbf{x}_{H,B})s_B(\omega)+rP^-$
P^-	$R(\mathbf{x},\mathbf{x}_A,\omega)s_A(\omega)$	$R(\mathbf{x},\mathbf{x}_B,\omega)s_B(\omega)$
Surface $\partial\mathcal{D}_m$		
P^+	$T(\mathbf{x},\mathbf{x}_A,\omega)s_A(\omega)$	$T(\mathbf{x},\mathbf{x}_B,\omega)s_B(\omega)$
P^-	0	0

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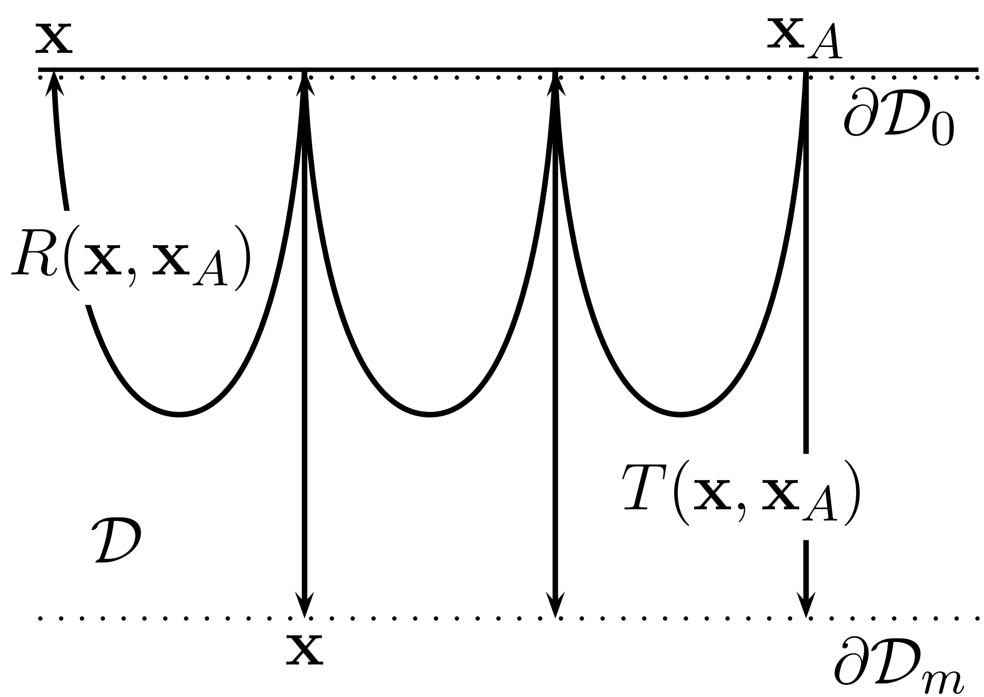
Passive Seismic

$$\delta(\mathbf{x}_{H,B} - \mathbf{x}_{H,A}) - 2\mathcal{R}[R(\mathbf{x}_A, \mathbf{x}_B)] = \int_{\partial\mathcal{D}_m} T^*(\mathbf{x}_A, \mathbf{x}) T(\mathbf{x}_B, \mathbf{x}) d^2\mathbf{x}$$



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R 2 T

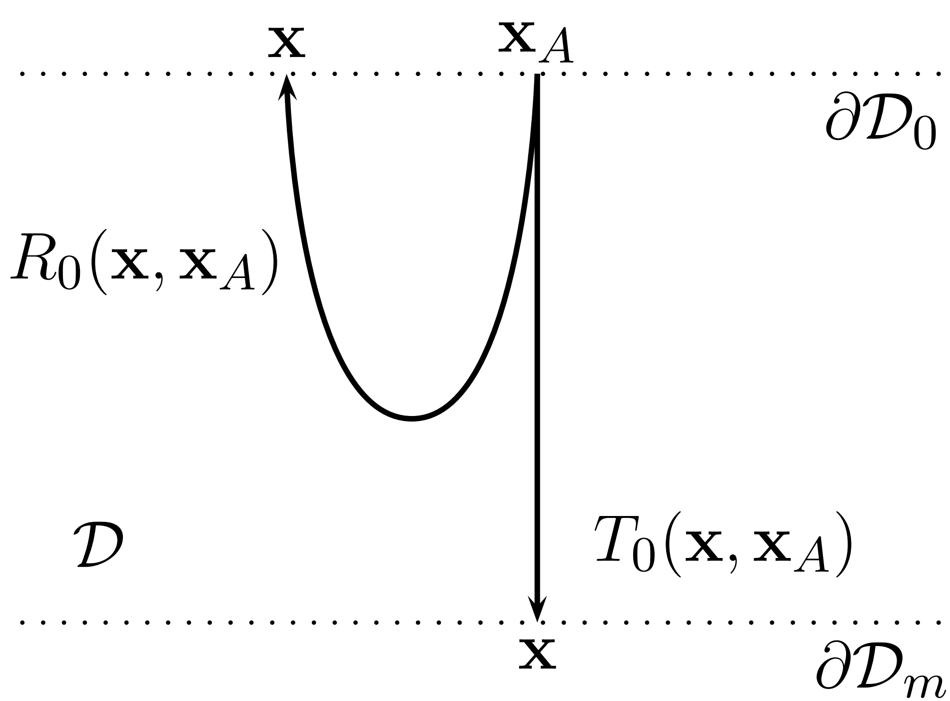
$$\int_{\partial\mathcal{D}_0} \{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\} d^2\mathbf{x} = \int_{\partial\mathcal{D}_m} \{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\} d^2\mathbf{x}$$

Surface $\partial\mathcal{D}_0$		
Field	**State A**	**State B**
P^+	$\delta(\mathbf{x}_H - \mathbf{x}_{H,A}) s_A(\omega)$	$\delta(\mathbf{x}_H - \mathbf{x}_{H,B}) s_B(\omega)$
P^-	$R_0(\mathbf{x}, \mathbf{x}_A, \omega) s_A(\omega)$	$R_0(\mathbf{x}, \mathbf{x}_B, \omega) s_B(\omega)$
Surface $\partial\mathcal{D}_m$		
P^+	$T_0(\mathbf{x}, \mathbf{x}_A, \omega) s_A(\omega)$	$T_0(\mathbf{x}, \mathbf{x}_B, \omega) s_B(\omega)$
P^-	0	0

R 2 T

$$\delta(\mathbf{x}_{H,B} - \mathbf{x}_{H,A}) - \int_{\partial\mathcal{D}_0} R_0^*(\mathbf{x}, \mathbf{x}_A) R_0(\mathbf{x}, \mathbf{x}_B) d^2\mathbf{x} =$$

$$\int_{\partial\mathcal{D}_m} T_0^*(\mathbf{x}, \mathbf{x}_A) T_0(\mathbf{x}, \mathbf{x}_B) d^2\mathbf{x}$$



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1D medium, 2D world

3 layer medium 1000-4000-1000 m/s thickness 200 m:
$4000/400 = 0.1$ s. internal multiple train.

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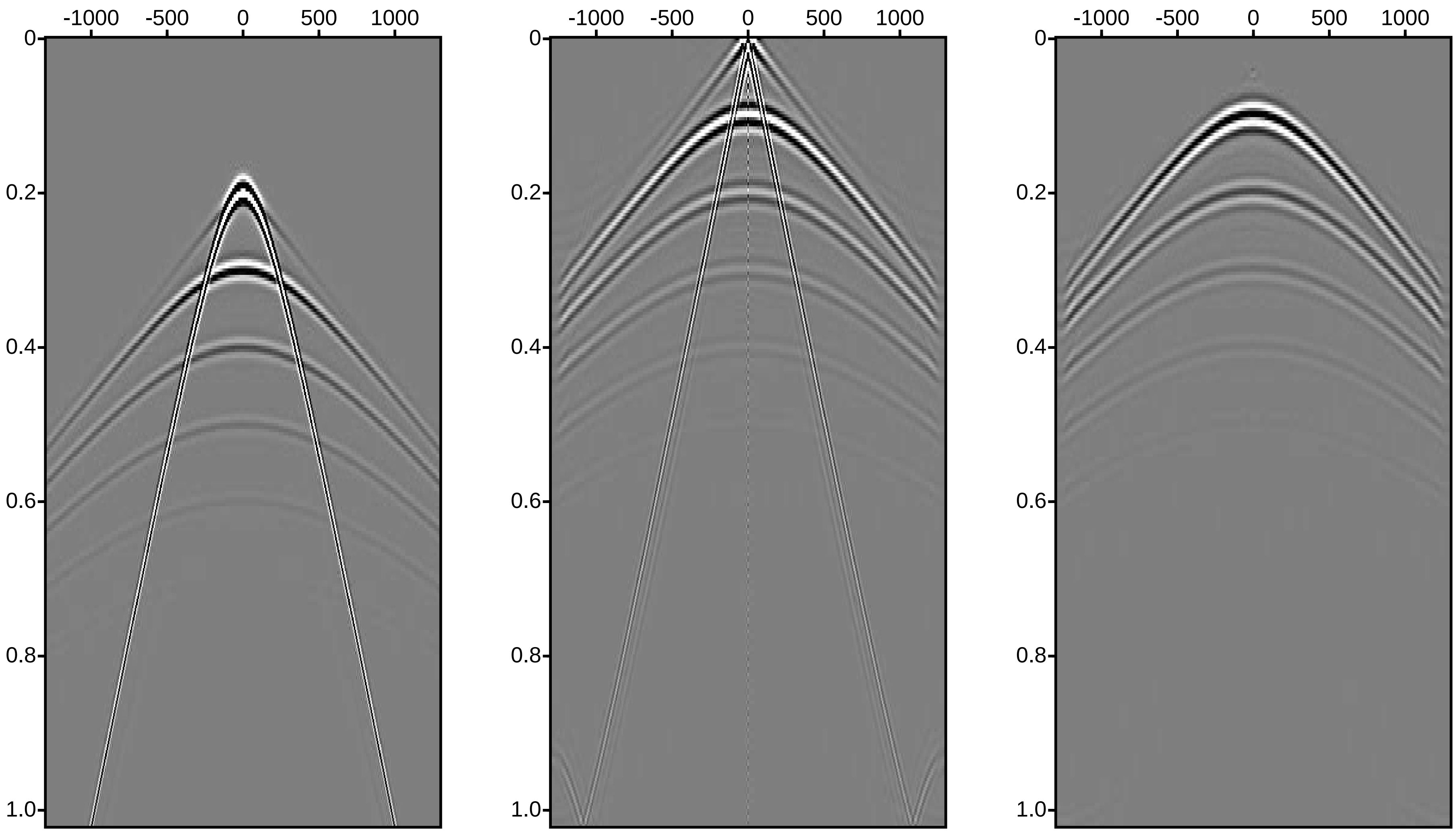
Comparison



$\boldsymbol{R}_0$ $\quad$ $\boldsymbol{R}_0^H \mathbf{R}_0$ $\quad$ $\mathbf{I} - \boldsymbol{R}_0^H \mathbf{R}_0$ muted

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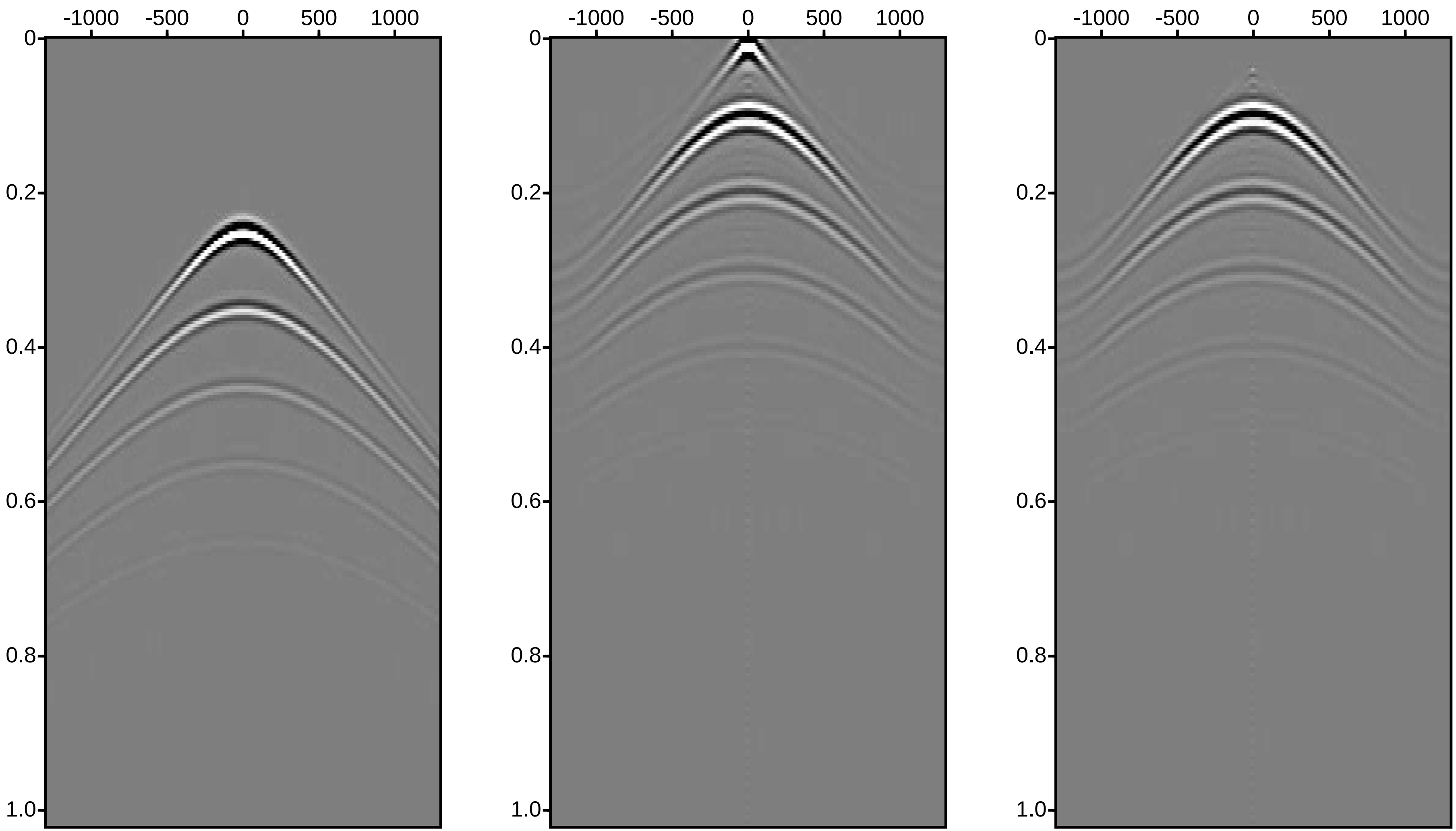
Comparison



$\boldsymbol{T}_0$ $\quad$ $\boldsymbol{T}_0^H \mathbf{T}_0$ $\quad$ $\boldsymbol{T}_0^H \mathbf{T}_0$ muted

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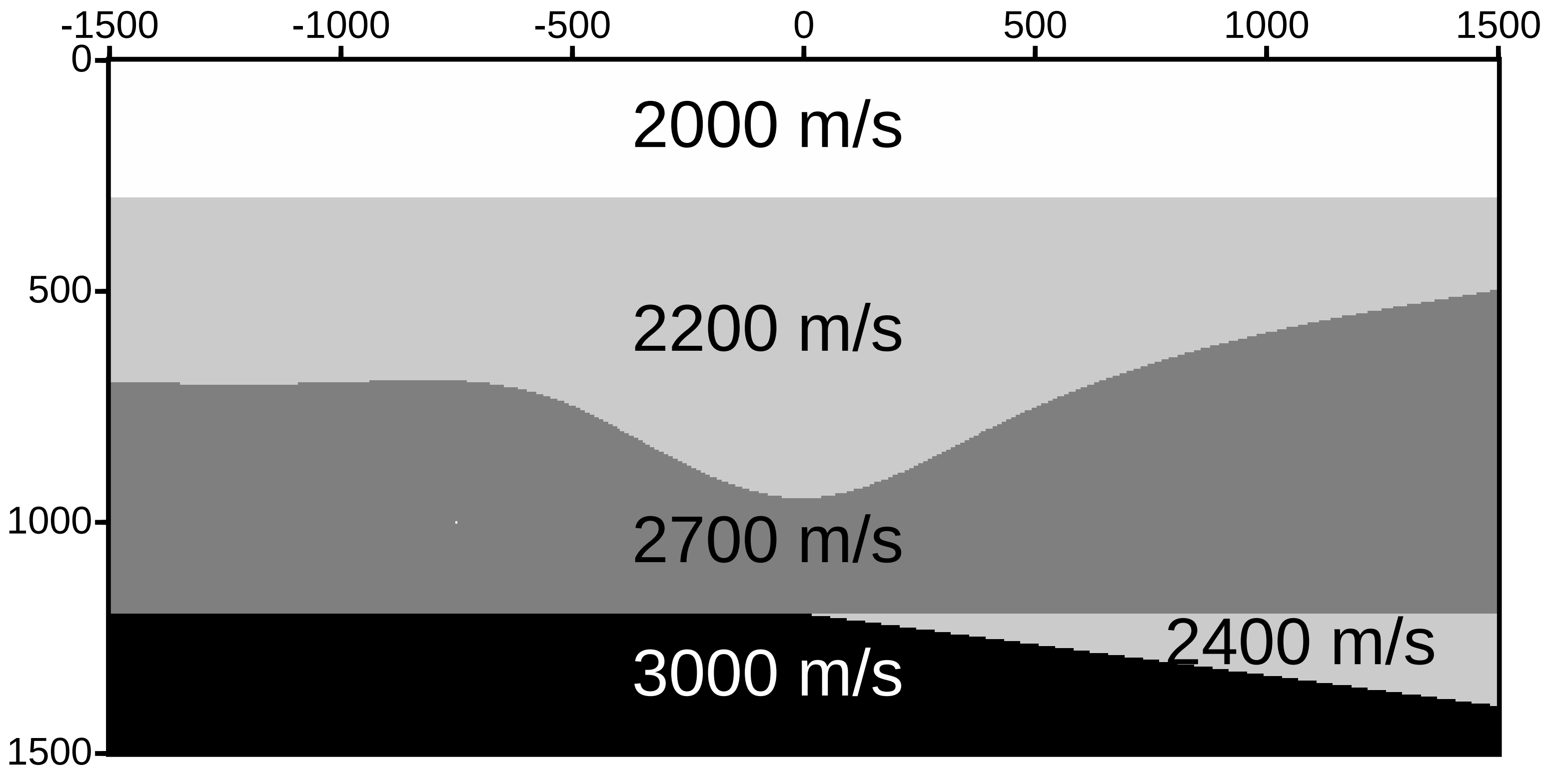
Syncline model



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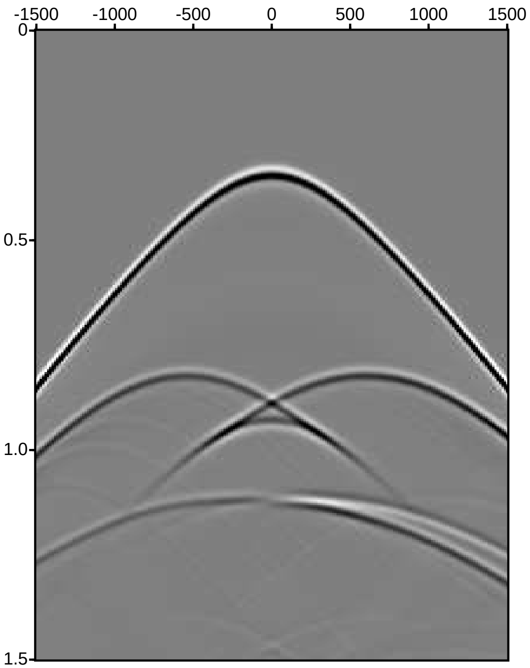
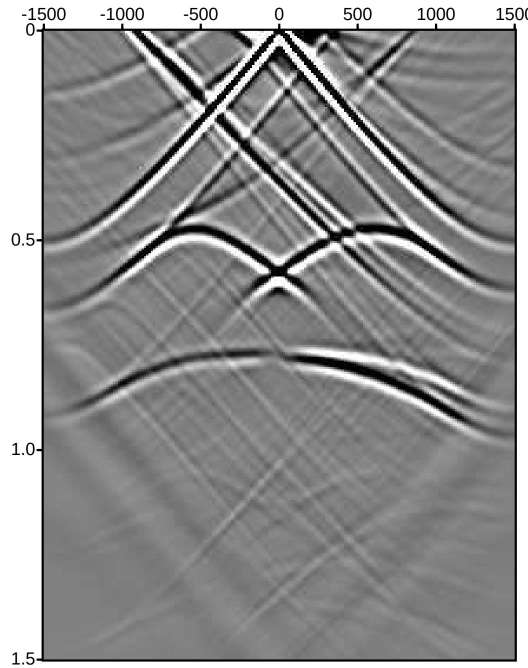
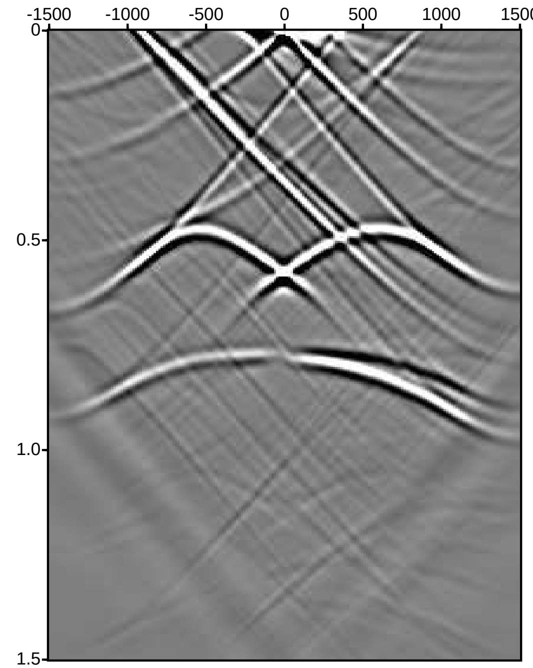
Comparison



$\boldsymbol{R}_0$

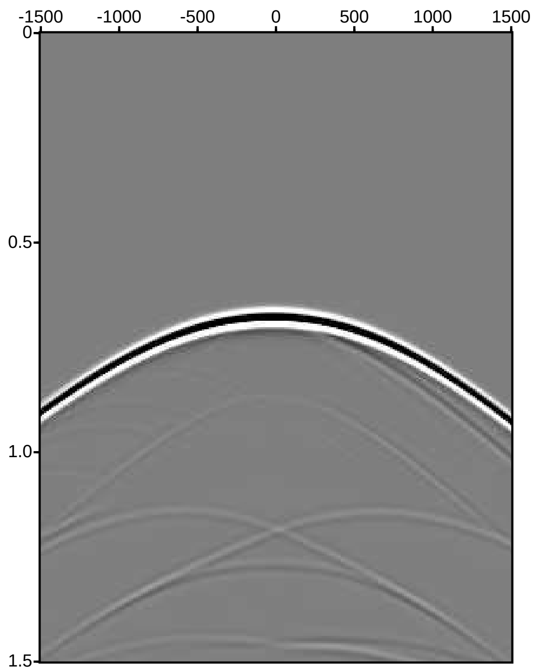


$\boldsymbol{R}_0^H \mathbf{R}_0$

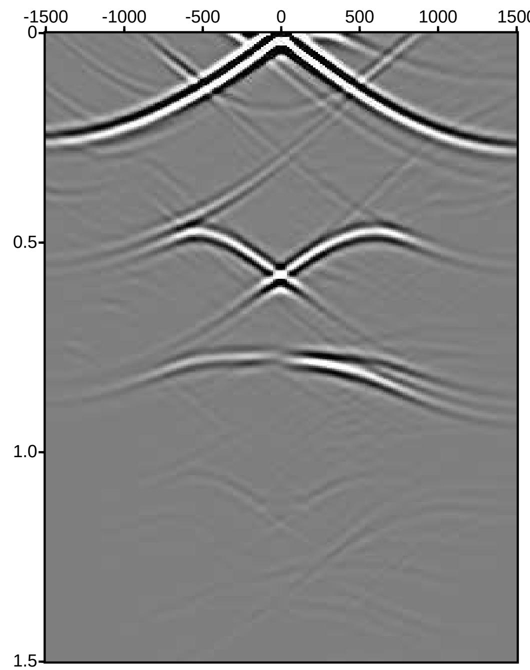


$\mathbf{I} - \boldsymbol{R}_0^H \mathbf{R}_0$

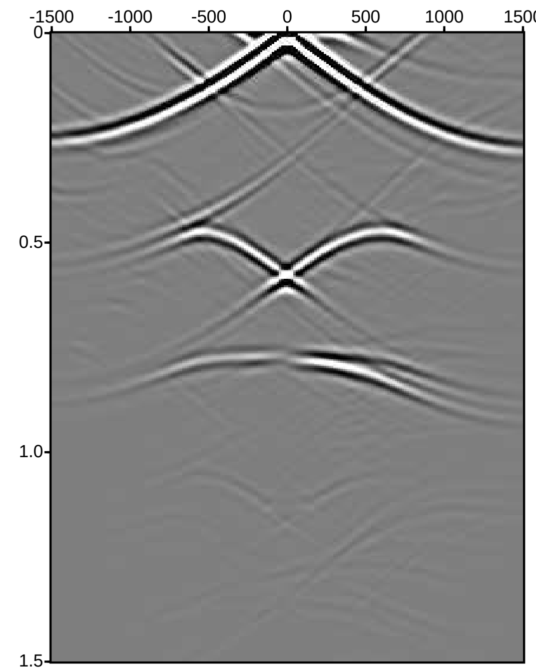
Comparison



$\boldsymbol{T}_0$



$\boldsymbol{T}_0^H \mathbf{T}_0$



$\boldsymbol{T}_0^H \mathbf{T}_0$

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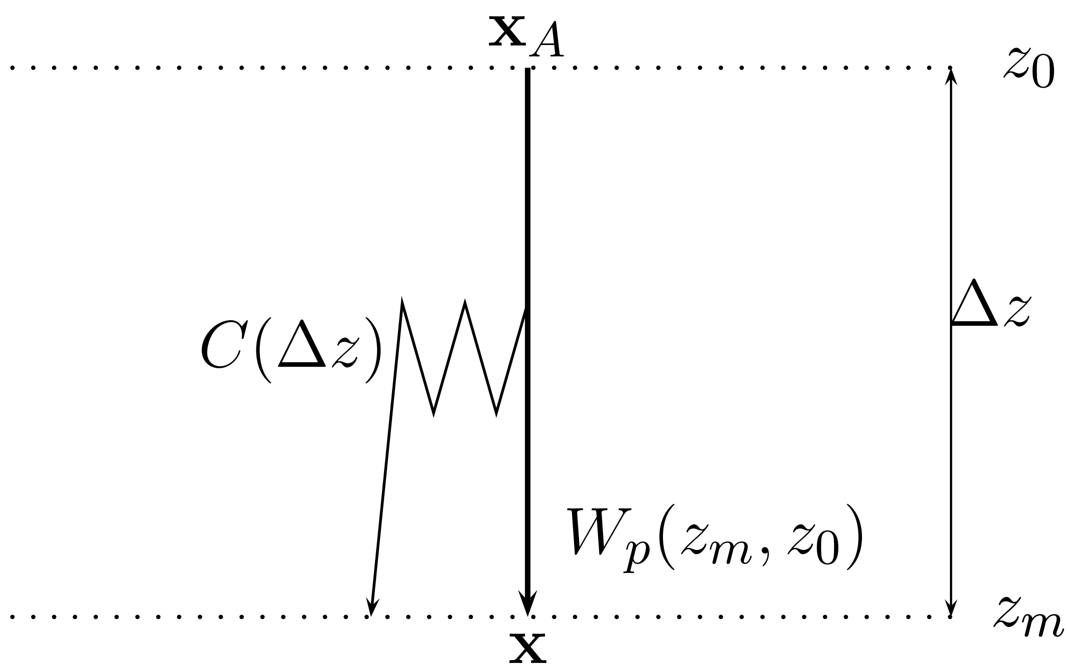
Model



$$\mathbf{T}_0(z_m, z_0) = \mathbf{W}_p(z_m, z_0)\mathbf{C}(\Delta z)$$

$$\mathbf{T}_0^H \mathbf{T}_0 = (\mathbf{W_p C})^H \mathbf{W_p C} = \mathbf{C}^H \mathbf{C} = \mathbf{I} - \mathbf{R}_0^H \mathbf{R}_0$$

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Assumptions (O'Doherty and Anstey)

$$C(p, \mathbf{x}_A, \Delta z) = \exp(-\mathcal{A}(p)\Delta z)$$

$$\mathbf{C} = \mathbf{L}\boldsymbol{\Lambda}_c\mathbf{L}^H$$

where

$$\boldsymbol{\Lambda}_c = \exp\{-\mathbf{A}\} = \begin{pmatrix} e^{-\mathcal{A}(\omega,p_1,\Delta z)} & 0 & \dots & 0 \\ 0 & e^{-\mathcal{A}(\omega,p_2,\Delta z)} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & e^{-\mathcal{A}(\omega,p_N,\Delta z)} \end{pmatrix}$$

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Detour: Matrix structures

For plane waves in 1D media **C** is a circulant matrix which has the property that its Fourier transform is equal to its eigenvalues:

$$\boldsymbol{\Lambda}_c = \mathcal{F}_{x \rightarrow k_x}\{\mathbf{C}\}$$

$$\mathbf{C} = \mathbf{F}^H \boldsymbol{\Lambda}_c \mathbf{F}$$

For non-plane waves and/or 2D media the eigenvalues are computed using numerical routines from LAPACK (zgeev, zheevx).

Eigenvalues of Matrix

Circulant (or Toeplitz) use FFT to calculate the eigenvalues:

$$\mathbf{C} = \begin{pmatrix} c_0 & c_{n-1} & c_{n-2} & \cdots & c_1 \\ c_1 & c_0 & c_{n-1} & \cdots & c_2 \\ c_2 & c_1 & c_0 & \cdots & c_3 \\ \vdots & \vdots & \vdots & & \vdots \\ c_{n-1} & c_{n-2} & c_{n-3} & \cdots & c_0 \end{pmatrix}$$

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Eigenvalues of Matrix

An $m \times n$ Toeplitz matrix can be embedded in a circulant matrix of order $m+n$ or smaller.

$$
\mathbf{T} = \begin{pmatrix}
x_m & x_{m+1} & & \cdots & x_{m+n-1} \\
x_{m-1} & x_m & x_{m+1} & & \vdots \\
\vdots & x_{m-1} & x_m & \ddots & \vdots \\
\vdots & & x_{m-1} & \ddots & x_{m+1} \\
\vdots & & & \ddots & x_m \\
\vdots & & & & x_{m-1} \\
\vdots & & & & \vdots \\
x_1 & & & & x_n
\end{pmatrix}
$$

Eigenvalues of Matrix (end detour)

Example 3×3 Toeplitz

$$\mathbf{T} = \begin{pmatrix} x_3 & x_4 & x_5 \\ x_2 & x_3 & x_4 \\ x_1 & x_2 & x_3 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} x_3 & x_4 & x_5 & 0 & 0 & 0 & x_1 & x_2 \\ x_2 & x_3 & x_4 & x_5 & 0 & 0 & 0 & x_1 \\ x_1 & x_2 & x_3 & x_4 & x_5 & 0 & 0 & 0 \\ 0 & x_1 & x_2 & x_3 & x_4 & x_5 & 0 & 0 \\ 0 & 0 & x_1 & x_2 & x_3 & x_4 & x_5 & 0 \\ 0 & 0 & 0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ x_5 & 0 & 0 & 0 & x_1 & x_2 & x_3 & x_4 \\ x_4 & x_5 & 0 & 0 & 0 & x_1 & x_2 & x_3 \end{pmatrix}$$

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Computational scheme

$$\mathbf{C}^H\mathbf{C} = \mathbf{I} - \mathbf{R}_0^H\mathbf{R}_0$$

$$\mathbf{C}^H\mathbf{C} = \mathbf{I} - \mathbf{L}\mathbf{\Lambda}_r\mathbf{L}^H$$

$$\mathbf{L}\mathbf{\Lambda}_c^H\mathbf{\Lambda}_c\mathbf{L}^H = \mathbf{L}[\mathbf{I} - \mathbf{\Lambda}_r]\mathbf{L}^H$$

The eigenvalues of the cross correlation matrix have now to be mapped from wavenumber (eigenvalue number) to ray-parameter p. Then the following relation gives the real part of the causal filters:

$$\mathbf{\Lambda}_c^H\mathbf{\Lambda}_c = \exp\{-2\mathcal{R}\{\mathbf{A}\}\}$$

$$\exp\{-2\mathcal{R}\{\mathbf{A}\}\} = \mathbf{I} - \mathbf{\Lambda}_r$$

$$\mathcal{R}\{\mathbf{A}\} = -\frac{1}{2}\ln\{\mathbf{I} - \mathbf{\Lambda}_r\}$$

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Computational scheme

Using the Hilbert transform, the causal functions can be reconstructed from their real part, this gives $\mathcal{A}(p)$. Inserting these computed functions into equation

$$C(p, \mathbf{x}_A, \Delta z) = \exp(-\mathcal{A}(p)\Delta z).$$

Together with an estimation of the primary propagator the calculated coda can be used to calculate the transmission response $\mathbf{T}_0$ with

$$\mathbf{T}_0(z_m, z_0) = \mathbf{W}_p(z_m, z_0)\mathbf{C}.$$

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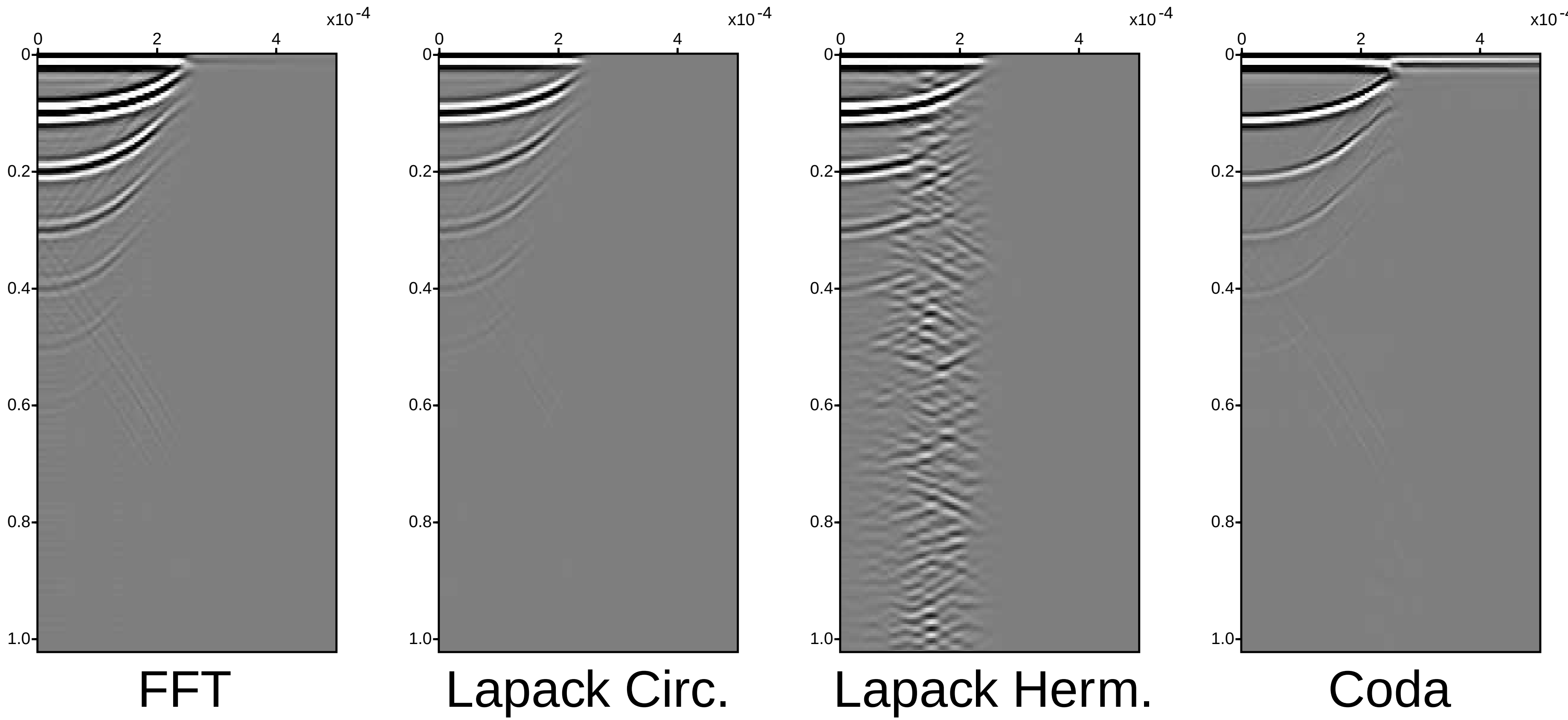
Calculated Eigenvalues in 1D media



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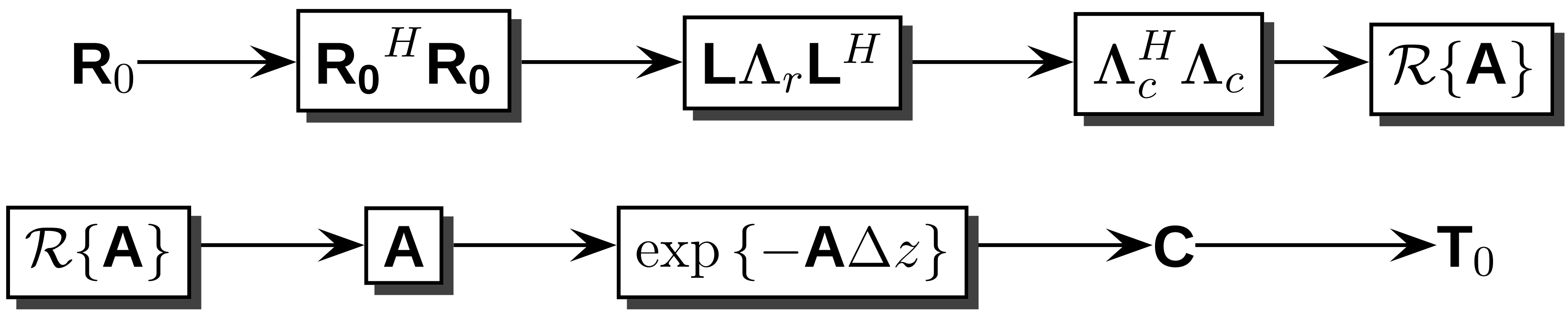
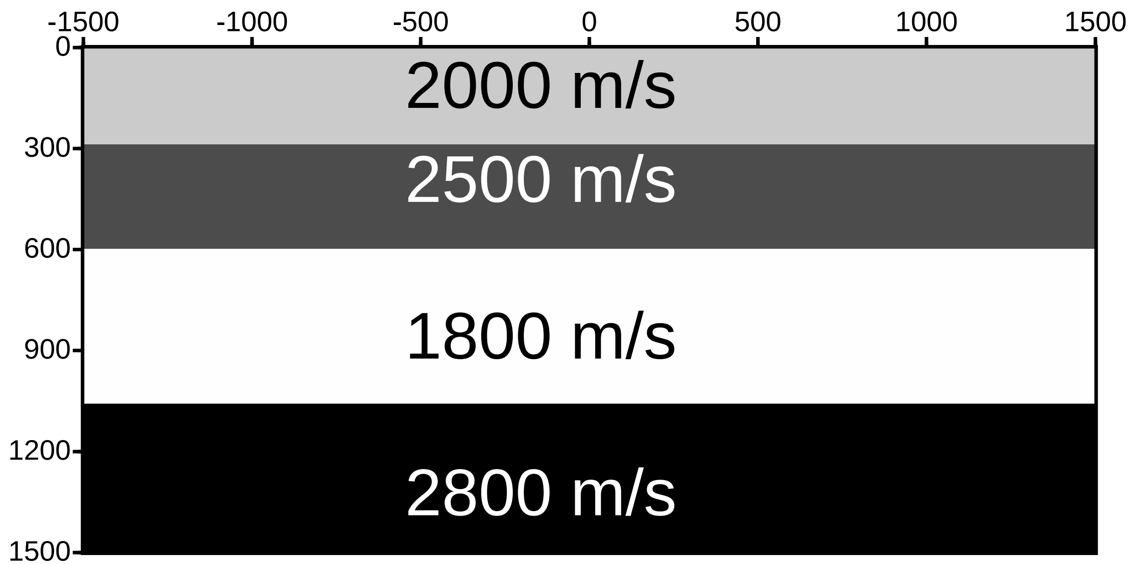
Scheme summary

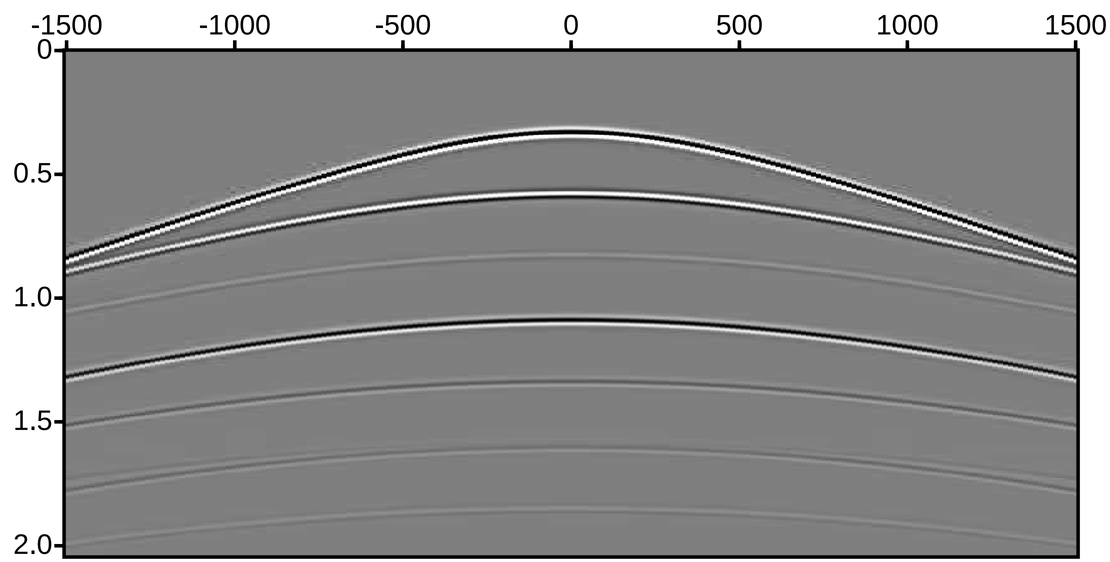
To summarize the procedure, the following steps must be taken to compute the transmission coda from reflection data:



where $\boldsymbol{\Lambda}_r$ contains the eigenvalues of $\mathbf{R}_0^H\mathbf{R}_0$ and $\mathbf{I}-\boldsymbol{\Lambda}_r=\boldsymbol{\Lambda}_c^H\boldsymbol{\Lambda}_c$.

Multi layer 1D model





shot record at x=0

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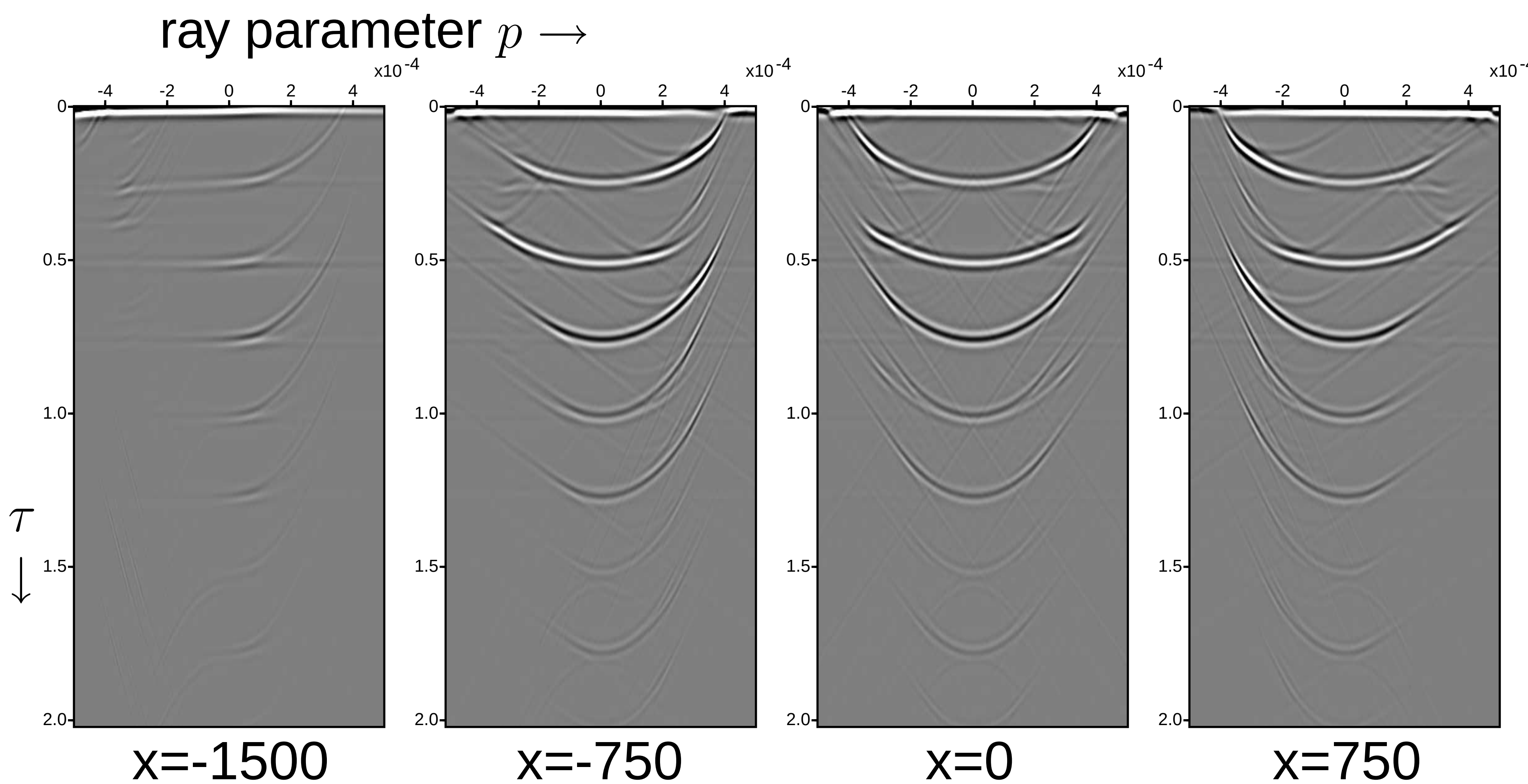
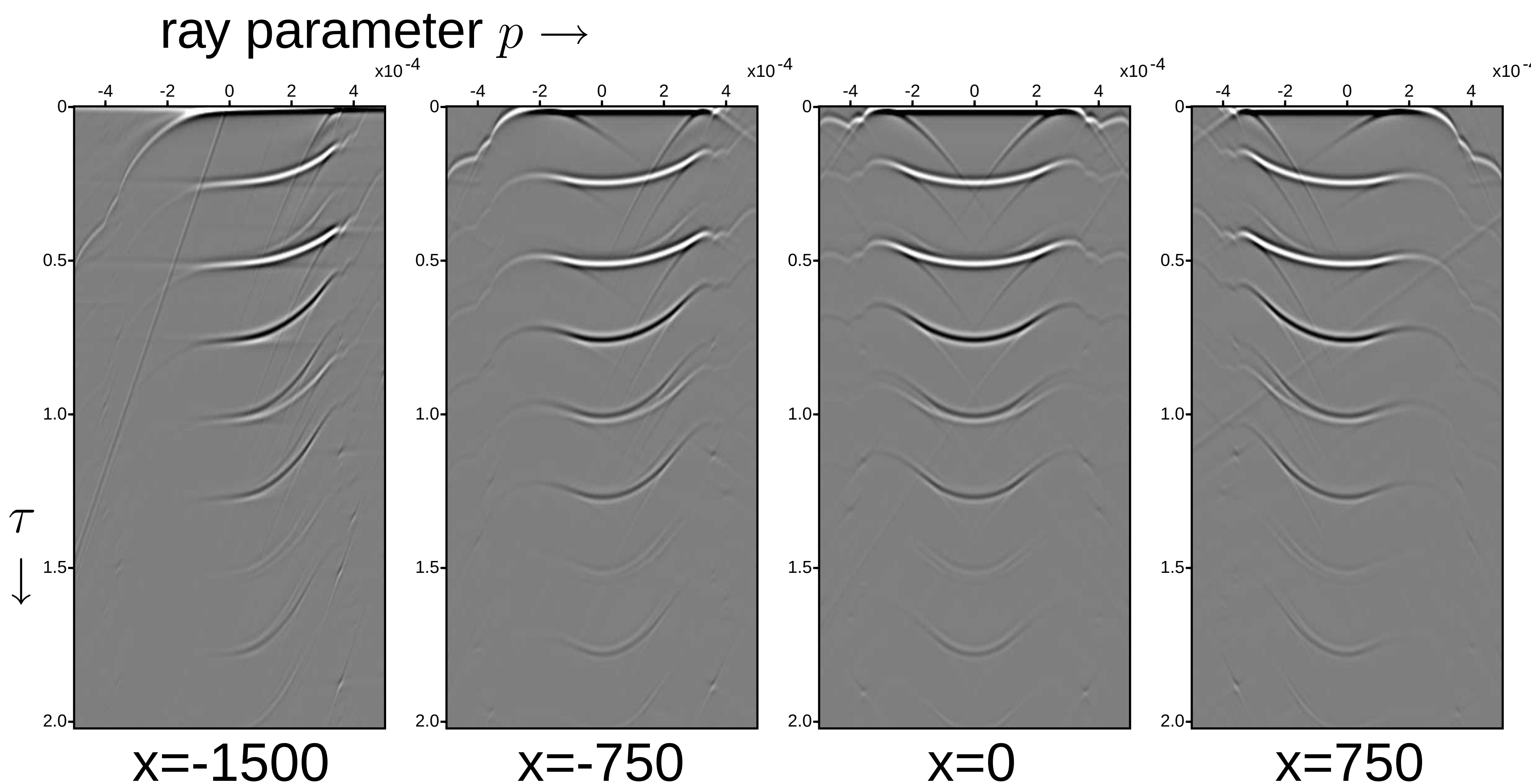
Multi layer 1D model



Calculated Eigenvalues using local 1D assumption

Multi layer 1D model



Modelled Transmission response

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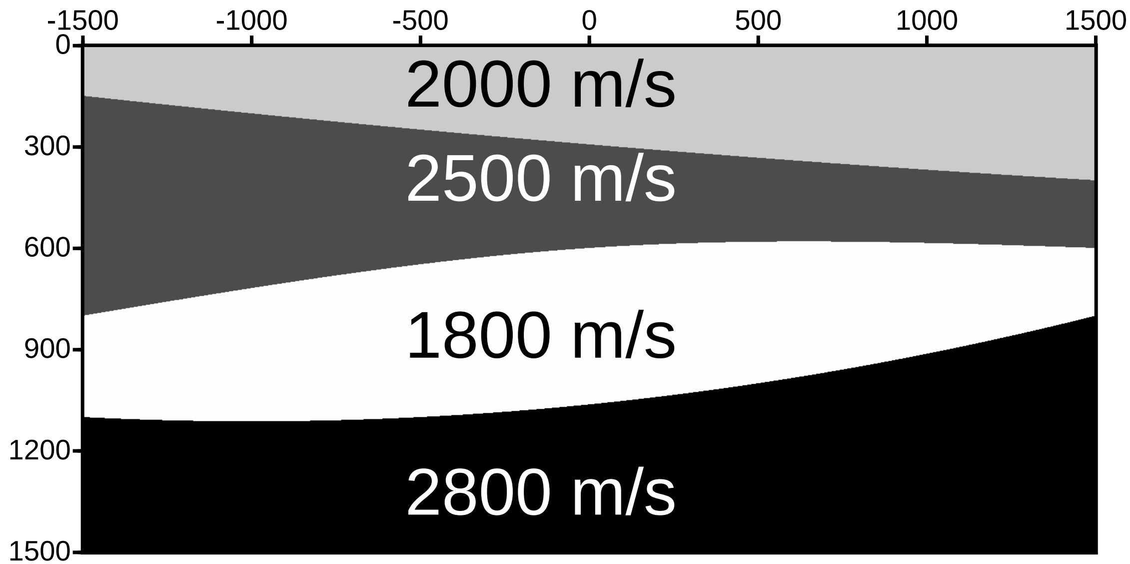
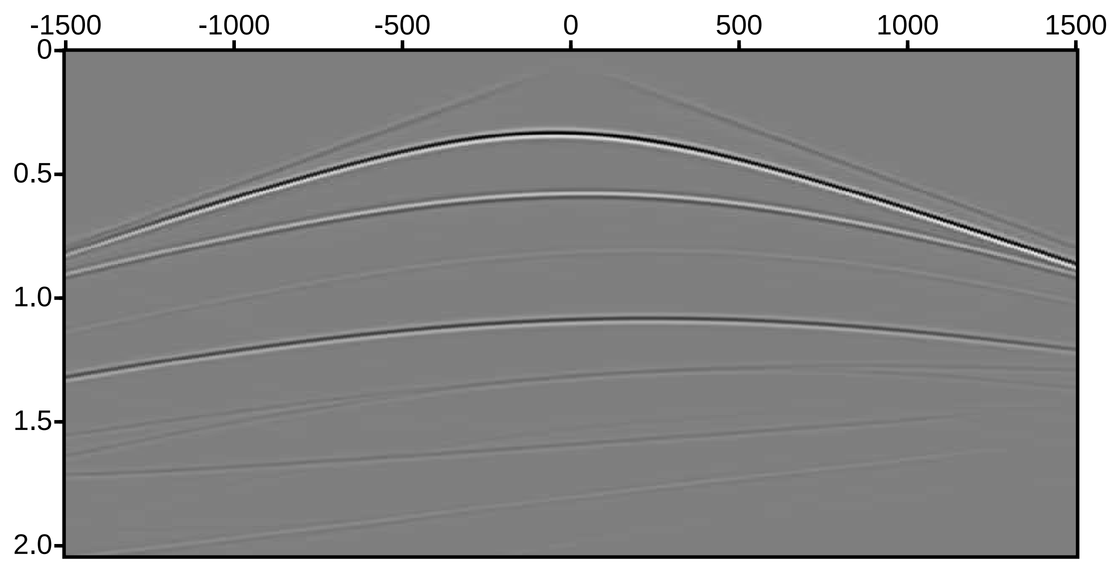
Multi layer simple 2D





shot record at x=0

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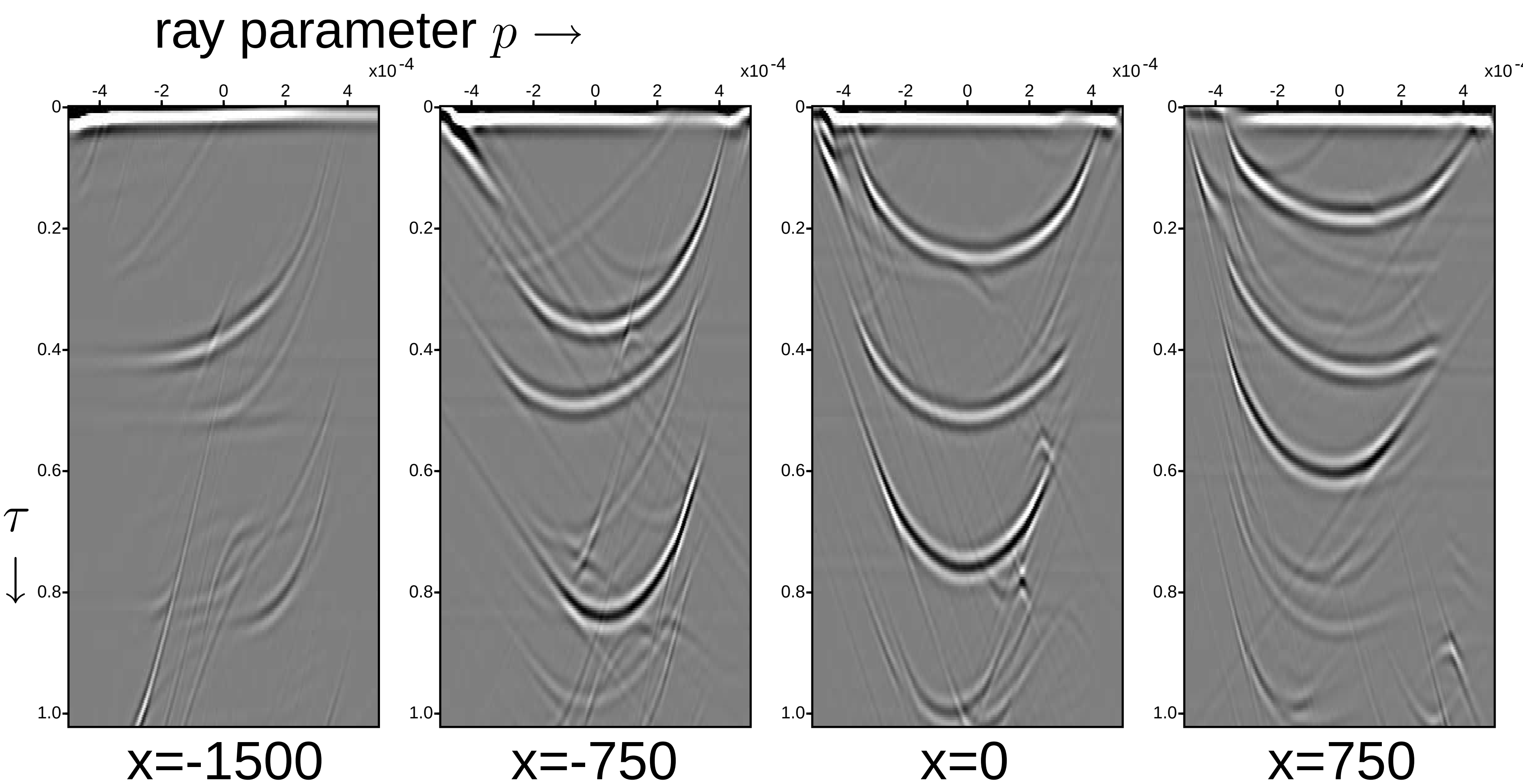
Multi layer simple 2D



Calculated Eigenvalues using local 1D assumption

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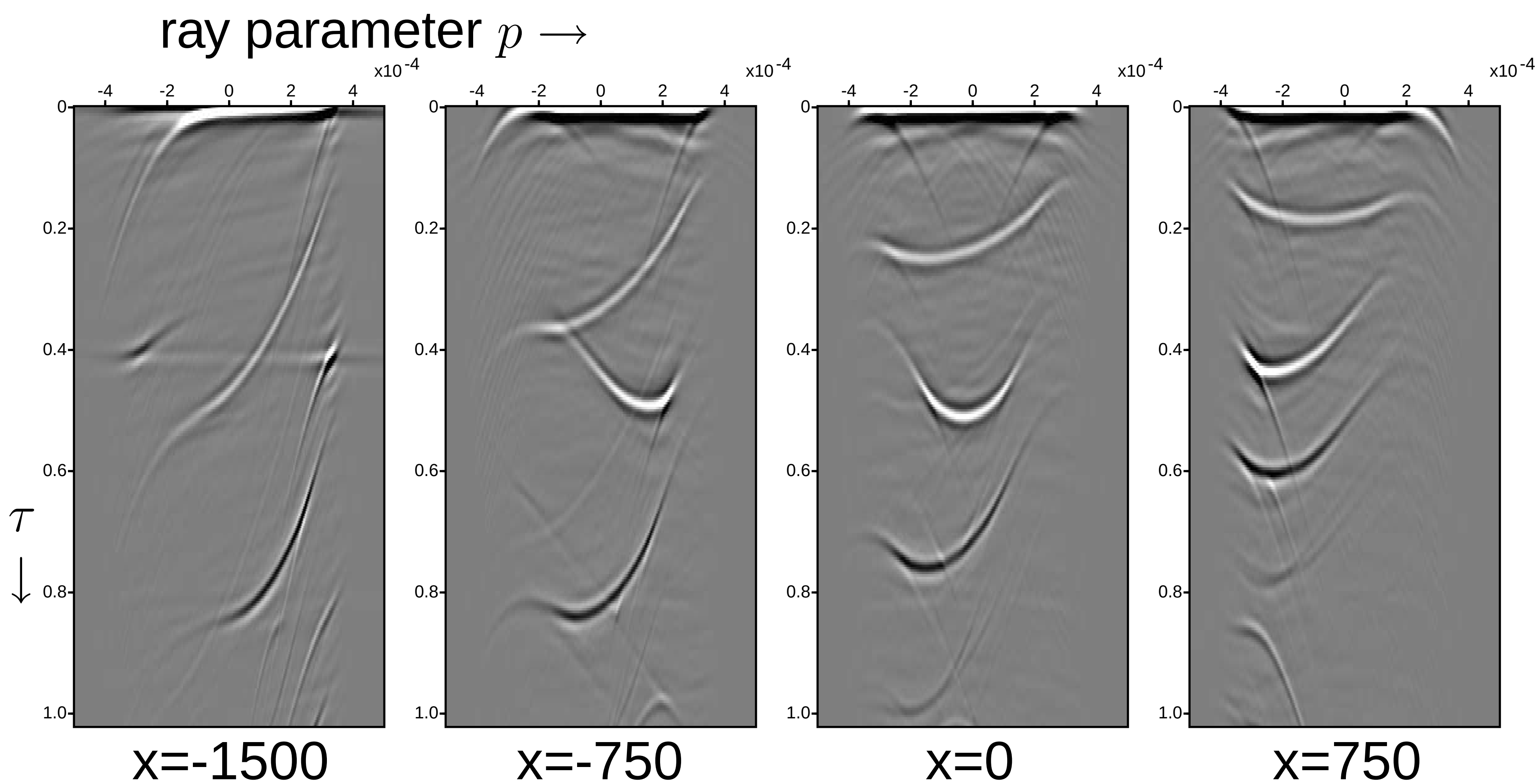
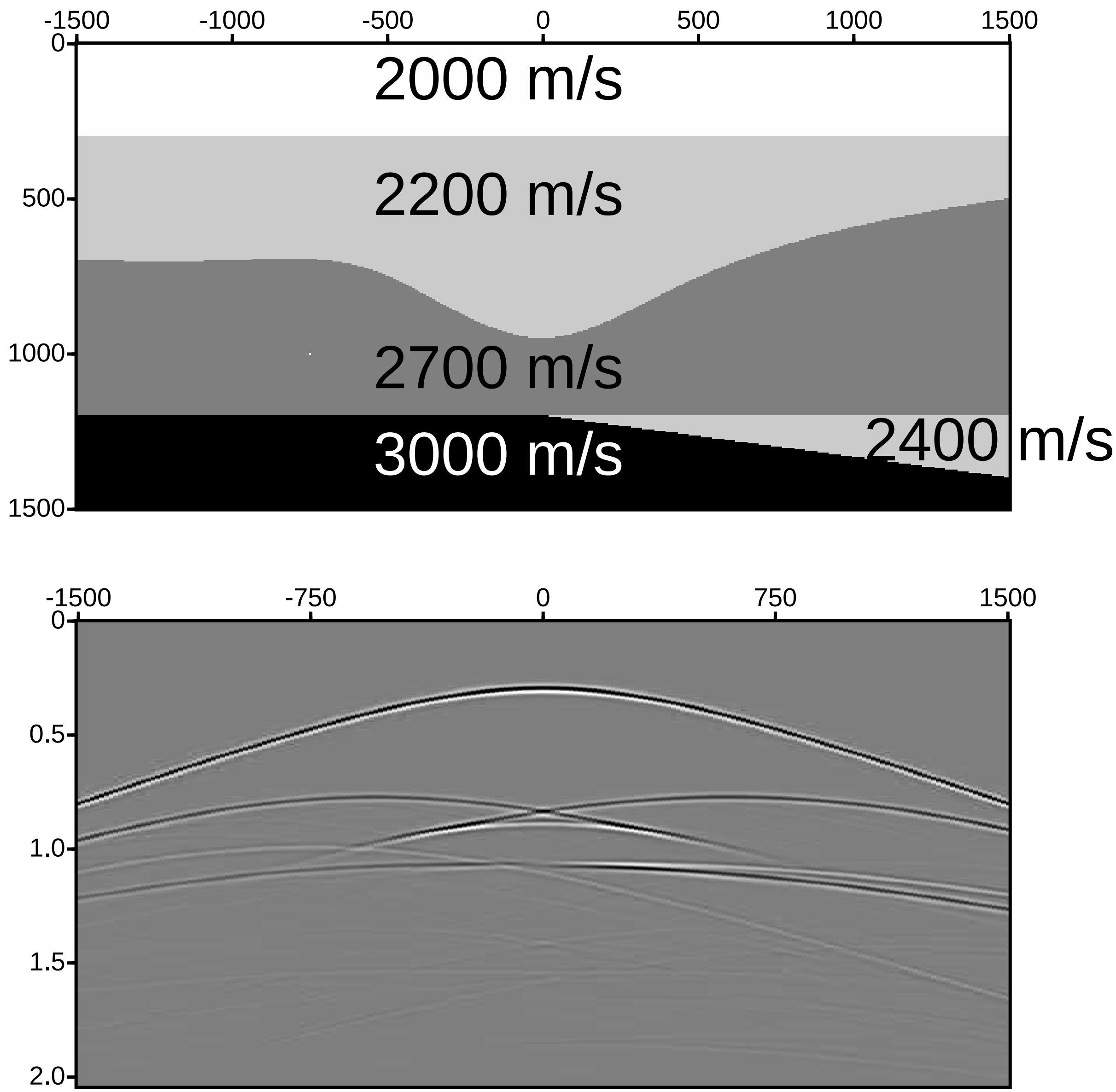
Multi layer simple 2D



Modelled Transmission response

Multi layer 2D



shot record at x=0

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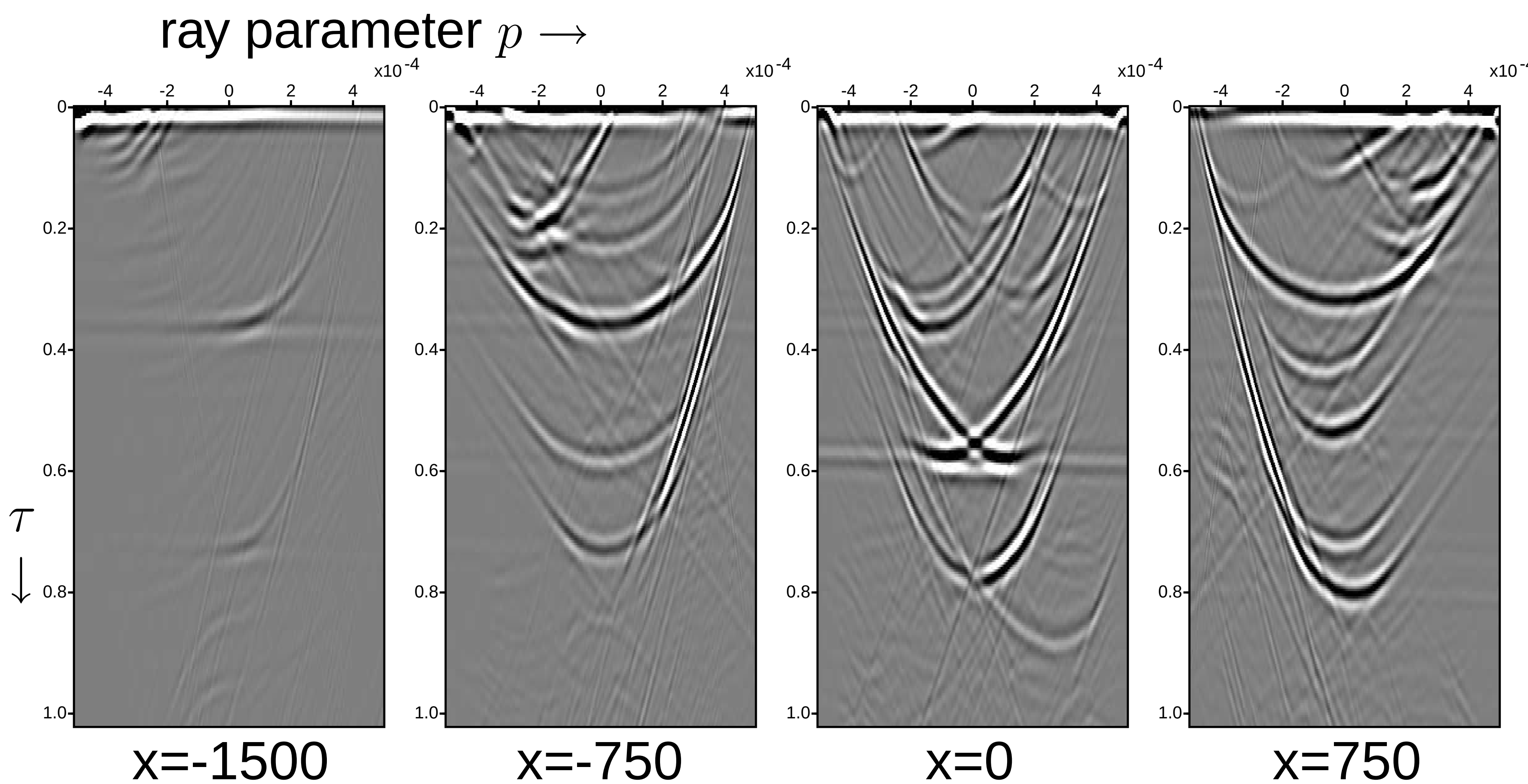
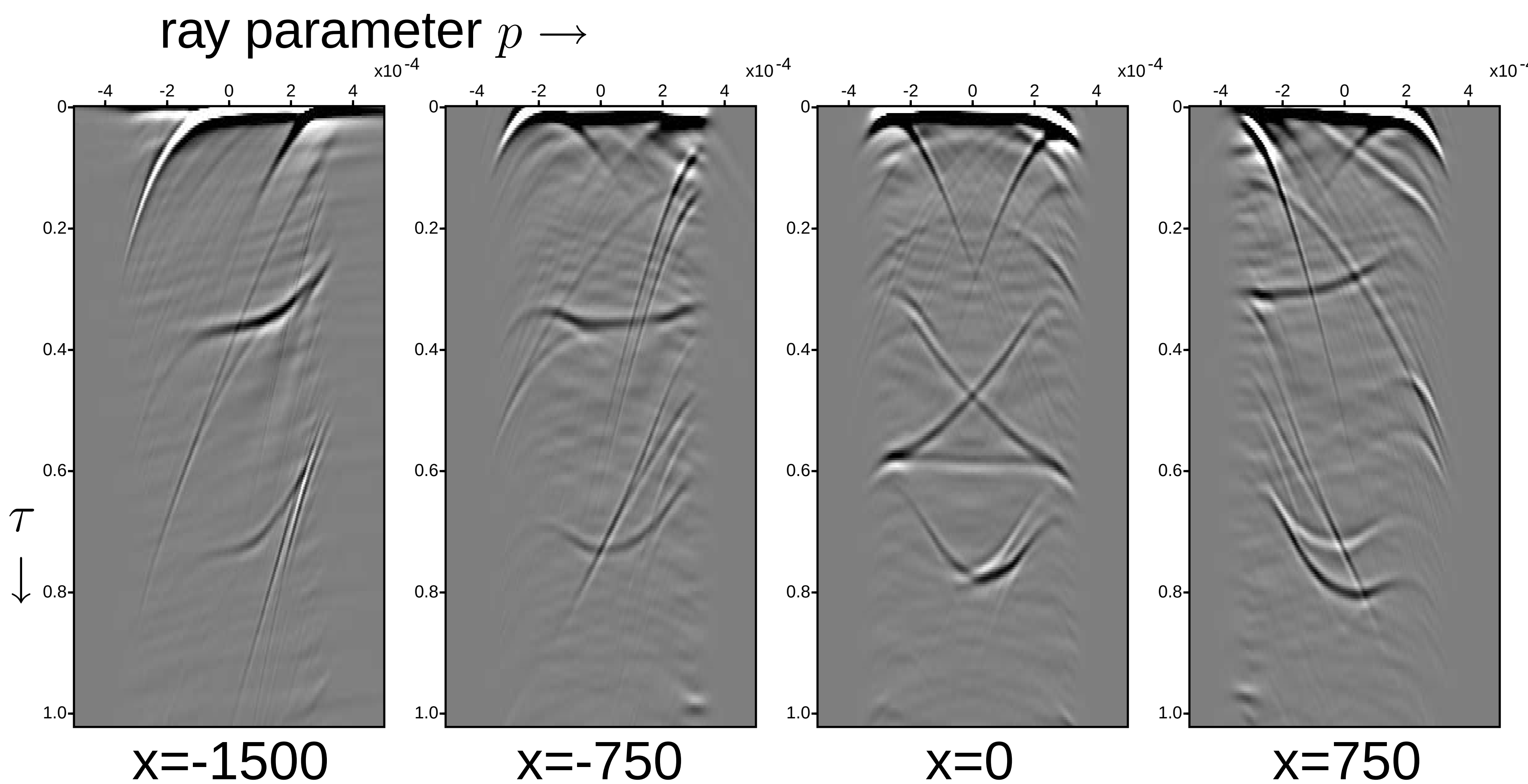
Multi layer 2D



Calculated Eigenvalues using local 1D assumption

Multi layer 2D



Modelled Transmission response

Conclusions

- Based on the one-way reciprocity theorem of the correlation type one can derive:
  - explicit relation for reflectivity from passive transmission data,
  - implicit relation for transmission from active reflection data.
- Correlated reflection panels contain information of the transmission coda.
- For 1D media this coda can be extracted
- For more complex media a local 1D assumption can be used to extract an first estimate of the coda

Acknowledgements

We would like to thank the research school ISES for supporting this research.

Downloads

Articles referred in this presentation:
http://www.xs4all.nl/~janth/Publications.html

This presentation can be found at:
http://www.xs4all.nl//~janth/Presentations/EAGE2006.pdf

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