Properties of Common Focus Point Gathers

Jan Thorbecke* and Scott Morton

Tuesday May 27, 1997

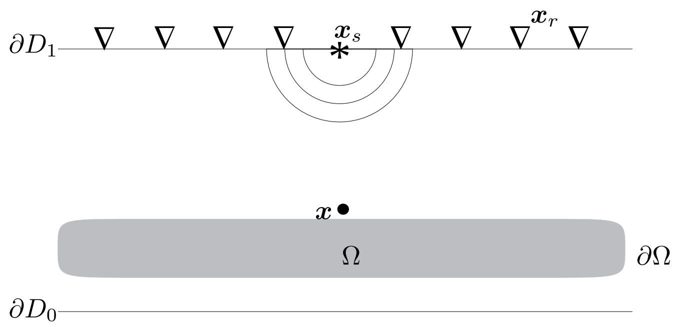
Slide 0

In January this year I received my PhD thesis from the University of Delft where I did my research on CFP gathers in the group of Professor Berkhout. In November 1996 I started to work in the Petroleum application group of Cray Reserach / Silicon Graphics, where I continue, together with Scott Morton to do research on the Common Focus Point gathers. In the previous presentations given this morning there was already a lot of work presented on CFP gathers. This presentation is about my works I have done at the University and aims at explaining some basic properties of CFP gathers. For the new people in this sessions I will briefly review the mathematics behind the CFP gather and will continue by showing (in a graphical and physical manner) how a CFP gather is constructed and what's so special about it.

Slide 1

Focusing integral for receiver array

$$P^{-,s}(\boldsymbol{x}, \boldsymbol{x}_s) = \int_{\partial D_1} W_p^{+,*}(\boldsymbol{x}, \boldsymbol{x}_r) P^{-,s}(\boldsymbol{x}_r, \boldsymbol{x}_s) \mathrm{d}^2\boldsymbol{x}_r,$$



Taking the configuration shown on the **slide** 1 into account it is observed there is only an upgoing scattered wave field at the surface ∂D_1. $W_p^{+,*}(\boldsymbol{x}, \boldsymbol{x}_r)$ represents the Green's function from $\boldsymbol{x}$ to $\boldsymbol{x}_r$and $P^{-,s}(\boldsymbol{x}_r, \boldsymbol{x}_s)$ represents the scattered field measured at $\boldsymbol{x}_r$ due to a source at $\boldsymbol{x}_s$. This equation is derived by using the one-way recipricity theorem as derived by Kees Wapenaar.

Physically $P^{-,s}(\boldsymbol{x}, \boldsymbol{x}_s)$ can be interpreted as a wave field obtained by backpropagating the scattered field $P^{-,s}(\boldsymbol{x}_r, \boldsymbol{x}_s)$ from the receiver locations $\boldsymbol{x}_r$ into the scattering region using the Greens function $G^*(\boldsymbol{x}, \boldsymbol{x}_r)$. An alternative interpretation is that it corresponds to focusing the receiver array (common shot gather) at an arbitrary image point inside the surface ∂D using the one-way Kirchhoff integral. Backpropagating the data from a receiver array as an intermediate step to imaging or inversion is well known in seismic migration and diffraction tomography. Making use of the redundancy present in the data, many shot locations, the same focusing procedure can also be used for the surface ∂D_1 at the positions where the sources are placed. Since we are dealing with discrete source and receivers positions at the surface the integral should be replaced by a summation and a matrix notation is more convenient.

Slide 2

Focusing matrix for receiver array

Focusing result:

$$\tilde{\boldsymbol{P}}_i^-(z_m,z_s) = \tilde{\boldsymbol{F}}_i^-(z_m,z_r)\mathbf{P}(z_r,z_s)$$

with operator

$$\tilde{\boldsymbol{F}}_i^-(z_m,z_r) \approx \tilde{\boldsymbol{I}}_i^-(z_m)\left[\mathbf{W}^+(z_m,z_r)\right]^*$$
$$\tilde{\boldsymbol{F}}_i^-(z_m,z_r)\mathbf{W}^-(z_r,z_m) = \tilde{\boldsymbol{I}}_i^-(z_m)$$

and forward model

$$\mathbf{P}(z_r,z_s) = \mathbf{W}^-(z_r,z_m)\mathbf{R}^+(z_m)\mathbf{W}^+(z_m,z_s)\mathbf{S}(z_s)$$

gives

$$\tilde{\boldsymbol{P}}_i^-(z_m,z_s) = \tilde{\boldsymbol{I}}_i^-(z_m)\mathbf{R}^+(z_m)\mathbf{W}^+(z_m,z_s)\mathbf{S}(z_s)$$

The used forward model of seismic reflection data, backscattered from one depth level at z_m, is given by

$$\mathbf{P}(z_r,z_s) = \mathbf{W}^-(z_r,z_m)\mathbf{R}^+(z_m)\mathbf{W}^+(z_m,z_s)\mathbf{D}^+(z_s)\mathbf{S}(z_s), \tag{1}$$

where z_r represents the receiver level, z_m the reflection level and z_s the source level. Focusing in emission can be regarded as a weighted summation (in phase and amplitude) along the common receiver arrays in such a way that the constructed wave front originates from a notional source at a point in the subsurface. The weighting operator used in this process is also called the focusing or synthesis operator, because the operator synthesizes the response of a focusing areal source from the seismic data. The principle of combining shot gathers at the surface for the synthesis of areal source responses, also referred to as areal shot record technology, was already introduced by Rietveld for controlled illumination in prestack depth migration. The synthesis operator for focusing in emission works on the rows of the matrix $\mathbf{P}(z_r,z_s)$ (common receiver array) and a synthesis operator for focusing in detection works on the columns of the matrix $\mathbf{P}(z_r,z_s)$ (common source array).

The focusing operator for the receiver array, applied to the left side of the right-hand side of equation (1), is defined as

$$\tilde{\boldsymbol{F}}_i^-(z_m,z_r)\mathbf{W}^-(z_r,z_m) = \tilde{\boldsymbol{I}}_i^-(z_m) \tag{2}$$
$$\tilde{\boldsymbol{F}}_i^-(z_m,z_r) \approx \tilde{\boldsymbol{I}}_i^-(z_m)\left[\mathbf{W}^+(z_m,z_r)\right]^*$$

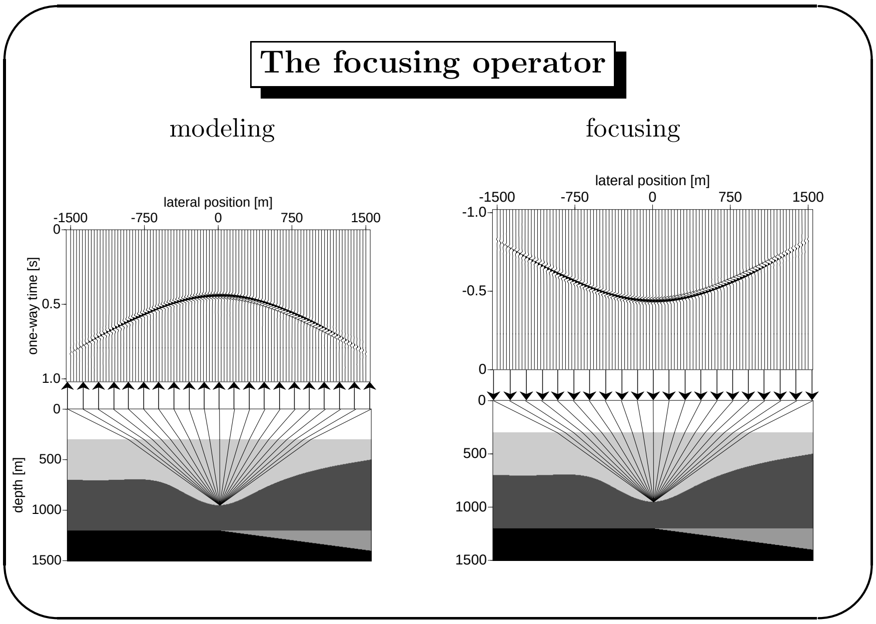
with $\tilde{\boldsymbol{I}}_i^-(z_m)$ a unit row vector with a 1 at the i^{th} position at depth z_m and $\tilde{\boldsymbol{F}}_i^-(z_m,z_r)$ the focusing operator acting at the receiver positions at the surface. Note that the approximation in equation (2) refers to the approximation of the inverse of the propagation operator $\mathbf{W}^-(z_r,z_m)$ by its matched filter $[\mathbf{W}^-(z_r,z_m)]^{-1} \approx [\mathbf{W}^+(z_m,z_r)]^*$

Substituting equation (2) into equation (1) gives an expression of the data after focusing of the detector array

$$\tilde{\boldsymbol{F}}_i^-(z_m,z_r)\mathbf{P}(z_r,z_s) = \tilde{\boldsymbol{P}}_i^-(z_m,z_s) = \tilde{\boldsymbol{I}}_i^-(z_m)\mathbf{R}^+(z_m)\mathbf{W}^+(z_m,z_s)\mathbf{D}^+(z_s)\mathbf{S}(z_s) \tag{3}$$

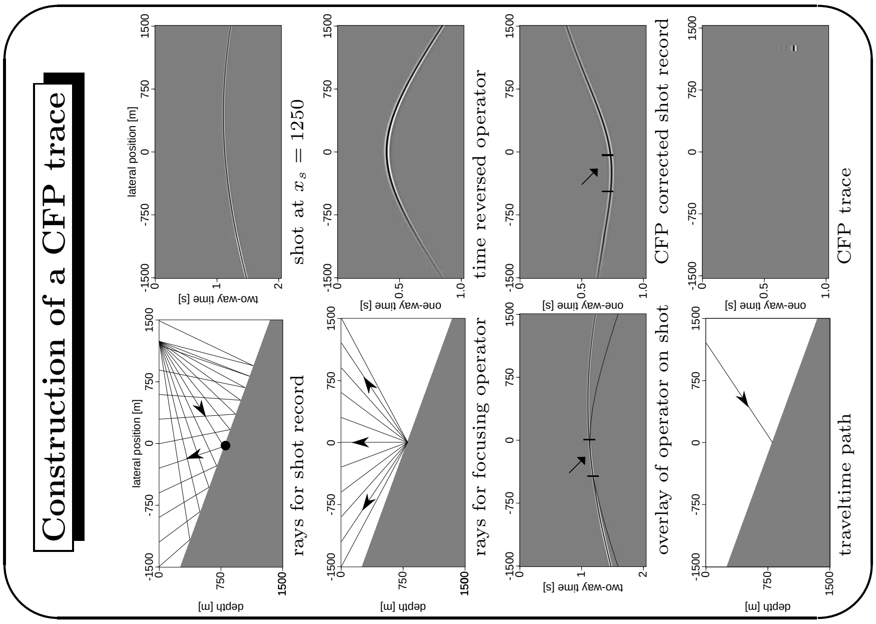
where equation (3) is an expression for the so called Common Focus Point (CFP) gather for focusing in *detection*.

Slide 3



To construct the CFP-gather from the data an initial synthesis (also called focusing) operator is needed. This initial synthesis operator can be based on stacking velocities or an initial macro model. The initial synthesis operator is calculated by positioning a point source at the desired grid-point in the subsurface followed by a forward modeling algorithm to calculate the source response at the surface. Measuring its response at the detector positions defines an operator for focusing in detection and measuring its response at the source positions defines an operator for focusing in emission. As shown in **slide** 3, the time reverse (complex conjugate in frequency domain) of the forward modeled response defines the synthesis operator.

Slide 4

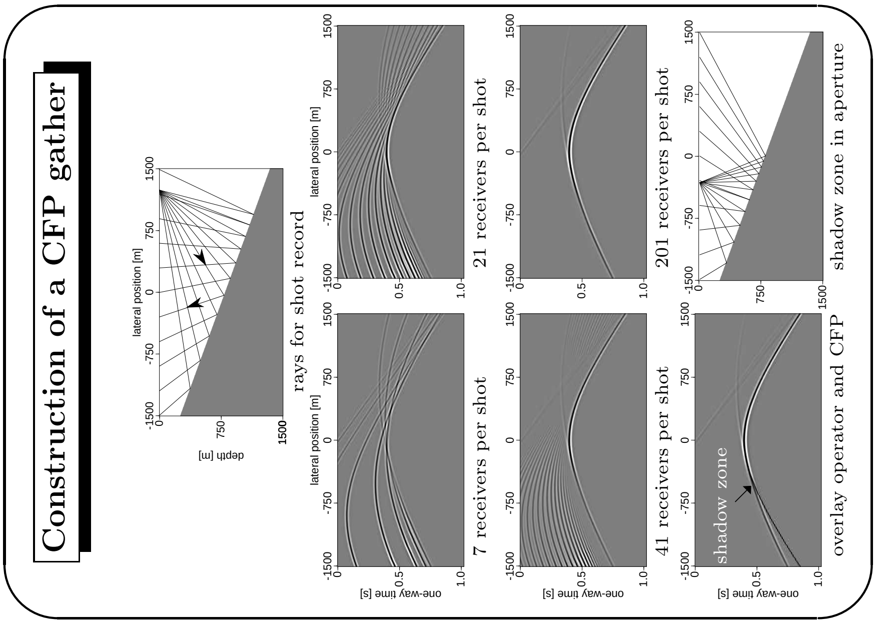


To calculate one trace out of a CFP gather for focusing in detection the synthesis operator measured at the detector positions must be convolved in time with a shot record followed by a summation along all traces in the record. This procedure is also shown by the integral of **slide** 1 and the matrix multiplication of **slide** 2. Performing this procedure, with the same operator for all shot records available defines the CFP gather for focusing in detection. Not all traces in the shot record have an equal contribution to the CFP trace, only that part of the CFP corrected shot record (the result after convolution with the operator but before summation of the traces) which lies in the Fresnel zone contributes to the CFP trace. The Fresnel zone represents that part of the shot record which is related to the chosen focus point and is therefore dependent on the focusing operator. The acquisition geometry of the shot record also plays a role because the measured data should at least contain reflection information from the defined focus point.

slide 4 shows how a CFP trace for focusing in detection is constructed by using a shot record and a focusing operator. Time convolution of the synthesis operator with the shot record gives the CFP corrected shot record. The arrow in the pictures indicate the Fresnel zone. The lateral position of the center of the Fresnel zone (the stationary phase point) can be determined by following the ray-path from the source, via the focus position, to the receiver position (indicated with an arrow). Another way to determine the stationary phase point can be done by superimposing the times of the operator over the shot record and shifting the operator times along the vertical axis until a part of the operator is tangent to the event in the shot record.

After summation over all the traces in the CFP corrected shot record the CFP trace is obtained. The only constructive 'interference' of the event in occurs at the Fresnel zone, which is indicated with an arrow. The constructed CFP trace is positioned in the CFP gather at the lateral position of the source in the shot record. The CFP trace is positioned at the source position because the time of the event in the CFP trace is equal to the time the wavefield needs to propagate from the source at the surface to the focus point at the reflector (g). This result is in agreement with equation (3) where it has been shown that one 'leg' of the two-way traveltime is compensated by the focusing operator in the first focusing step. By using more shot records more CFP traces can be constructed by using the same focusing operator and finally when all available shot records are used the construction of the CFP gather is completed

Slide 5

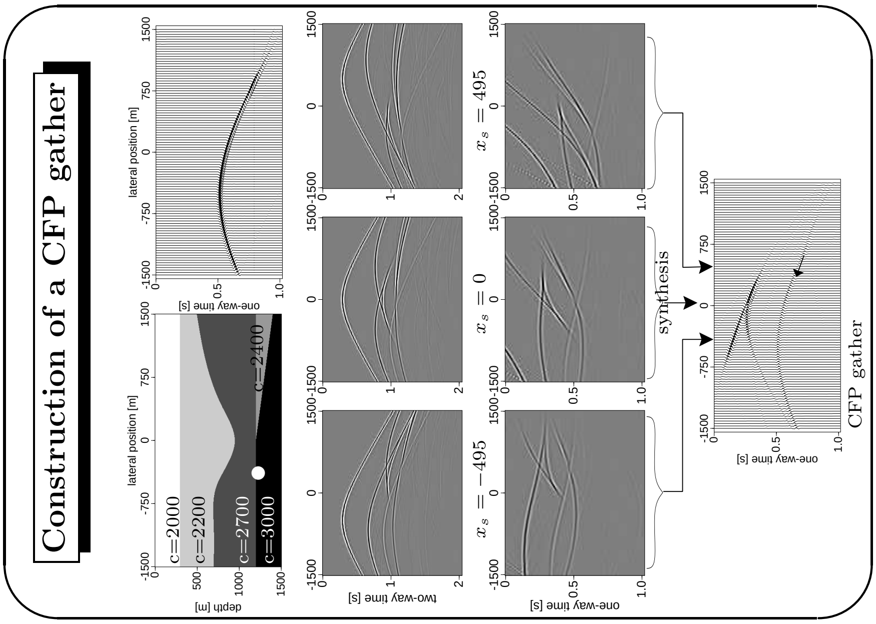


Another way to show how a CFP gather is built up from different shot records is shown in **slide** 5. The same operator and model configuration of **slide** 4 is used, but now all shots are used to construct the CFP gather (201 shots with a sampling distance equal to the (minimum) receiver spacing = 15 m) and the number of receivers is different for the different results shown in **slide** 5. In the top picture only 7 receiver per shot record are used in the construction of the CFP gather for focusing in detection. This means that the Fresnel zone is sparsely sampled and constructive interference does not occur after summation of only 7 traces. By looking at the picture the 7 contributions of the individual receivers can still be recognized. Using more receivers in the shot record shows how the Fresnel zone is built up in the CFP domain.

The bottom picture shows the complete CFP gather with the times of the synthesis operator super imposed on the result. Looking at the traveltimes of the operator and the focus point response it is observed that they are equal for most traces. This has to be expected because the time remaining in the CFP gather is the time from the defined focus point to the source positions (which is in fact the modeled operator). This important observation is called the principle of equal traveltime and states that if the focusing operator represents correct propagation in the model then in the time domain the result of the first focusing step $\tilde{\boldsymbol{P}}_i^-(z_m, z_s)$, represented by equation (3), is in traveltime equal to the time reversed focusing operator $\tilde{\boldsymbol{F}}_i^-(z_m, z_r)$.

However, by looking at the overlay it is observed that in the left part in the CFP gather, the times of the operator don't match with the times of the CFP response. This seems confusing at first sight, but considering the lateral position of the Fresnel zone for the shots positioned at the left side it is observed that the Fresnel zone falls outside the receiver aperture. So, the mismatch in time between the focusing operator and the CFP response is due to limited aperture. The zone where the Fresnel zone is 'shifted' outside the aperture is called the shadow zone. The ray paths of the shot record with its stationary phase ray just inside the receiver aperture is shown the right hand side bottom picture.

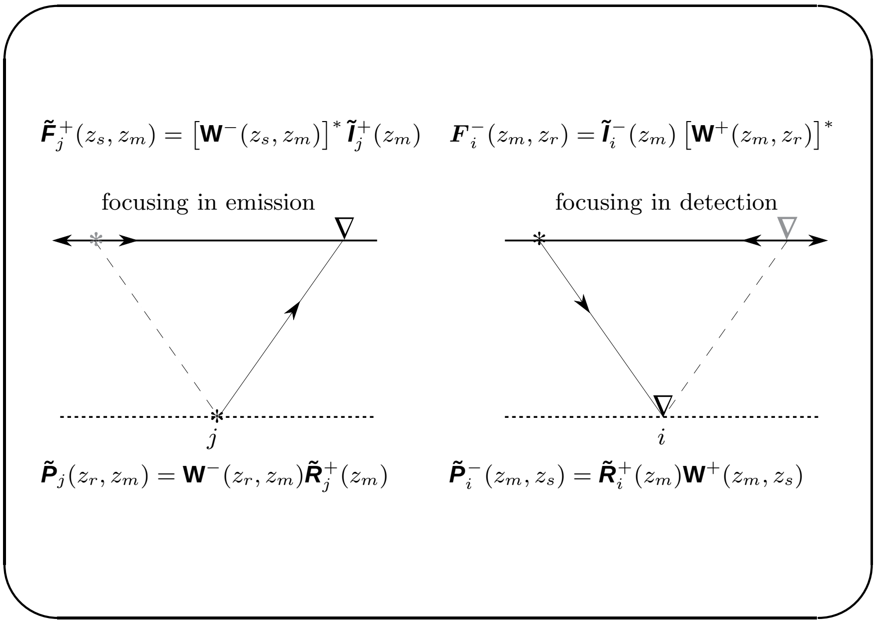
Slide 6



The construction of a CFP gather in a more complicated model is explained by using numerical data based on the model shown in **slide** 6. The numerical data is modeled with a fixed acquisition spread where the source positions are defined at every receiver position (201 shot positions with $\Delta x = 15$ m). The source has a dipole character and its signature is given by a Ricker wavelet with a frequency peak at 26.4 Hz. For the forward modeling of the data an acoustic finite difference algorithm is used. The subsurface model includes a diffraction point at $z = 1000, x = -750$ m and a negative reflection coefficient for the wedge located in the right corner of the model.

The synthesis process for a focusing receiver with a focus point defined at the synclinal interface at $x = 0$ and $z = 950$ m (the focus point is indicated with a black bullet) is shown in detail in **slide** 6. Th eshown focusing operator is applied to all common shot gathers available. Three different common shot gathers with source positions at $x = -495, x = 0$ and $x = 495$ m are used to illustrate the process. Convolution along the time axis of the traces in the shot gathers with the traces in the synthesis operator gives the intermediate synthesis results. Note that in these intermediate synthesis results the bow-tie of the syncline interface is still present. Summation over all the traces in the intermediate synthesis result defines one trace of the CFP gather. The most important contribution in the integrated result is determined by the Fresnel zone related to the focus point. If the focusing operator is correct then the operator time at the source position is identical with the time of the event present in the CFP trace. The summed trace is placed in the CFP gather at the position of the source. By carrying out the convolution and integration along the traces in the gather for all shot gathers available the CFP gather for focusing in detection is constructed. Only the deep reflector gives a contribution to the CFP response. Note that at the times before the CFP response a 'non-causal' event is observed, which originates from the syncline interface.

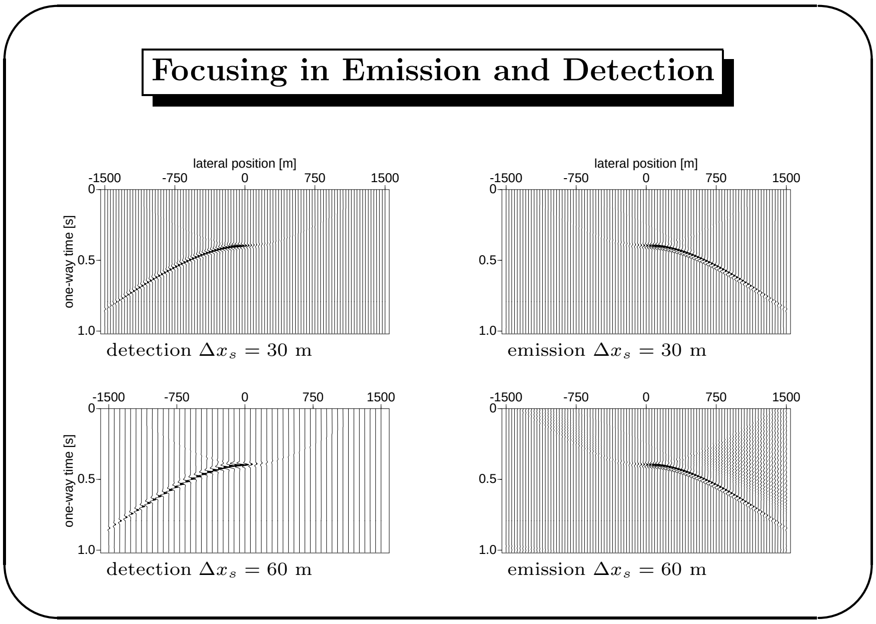
Slide 7



Focusing in detection positions a virtual receiver on a reflecting boundary and focusing in emission positions a virtual source on a reflecting boundary.

Note that if the backpropagating Green's function, which is expressed by $\tilde{\boldsymbol{F}}_i^-(z_m, z_r)$, represents correct propagation then the result of the first focusing step $\tilde{\boldsymbol{P}}_i^-(z_m, z_s)$, represented by equation (3), is in traveltime equal to the time reversed focusing operator $\tilde{\boldsymbol{F}}_j^+(z_s, z_m)$. Here the principle of equal traveltime is shown in matrix formulation. This principle plays a fundamental role in the updating procedure described later where a seismic image can be built up without knowing the background model. The principle can also be used to update the initial background model of the Green's function. Note that for a CFP gather designed for focusing in detection one should compare the traveltimes for an operator defined at the source position, which is the operator for focusing in emission.

Slide 8

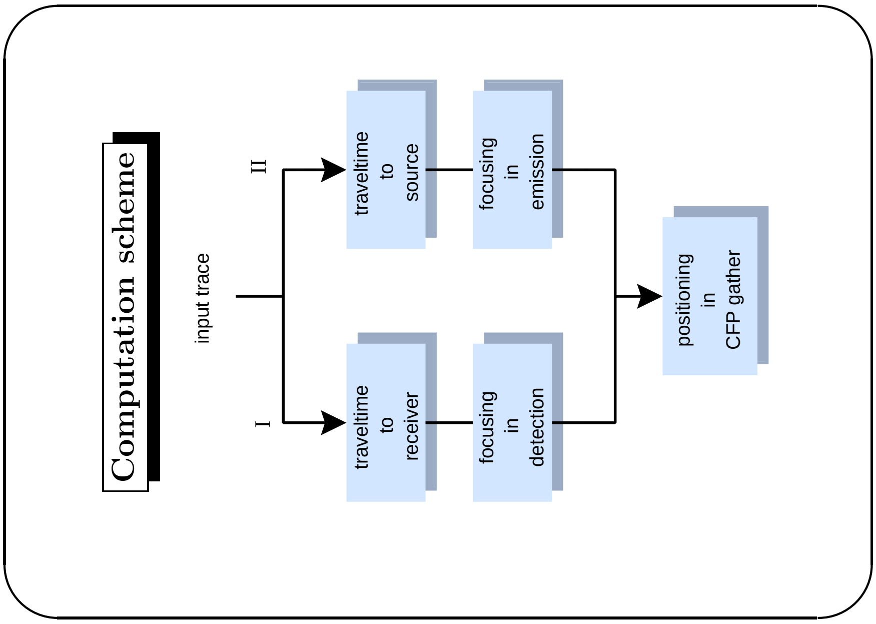


In all previous shown CFP gathers, focusing in detection was carried out in the first focusing step. The focusing operator was defined at the receiver positions and the traces in the CFP gather were defined at the source positions. However, it is also possible to do focusing in emission in the first focusing step. The focusing operator must then be defined at the source positions and the resulting CFP gather will have its traces positioned at the receiver positions.

To show clearly the differences between focusing in emission and focusing in detection a flat reflector at 800 m depth with a velocity above the reflector of 2000 m/s is chosen. The shot and receivers are moving above the reflector from left to right, the first receiver is positioned at the source position and the last receiver is positioned 3000 m to the right of the first receiver, the distance between the receivers is 30 m. The shots are positioned at 30 m from each other in the top pictures of **slide** 8 and at 60 m in the bottom pictures. Comparing both pictures for emissiona nd detection it is observed that due to the marine type of acquisition geometry the focus is illuminated differently for focusing in detection and focusing in emission.

Focusing in detection gives a CFP gather sampled with traces 60 m (the shot distance) from each other. Every trace is built from a shot record with the sampling distance of the receivers. In the result for focusing in emission the CFP trace spacing is 30 m (the receiver distance). Every trace in this CFP gather is built from a common receiver gather which is sampled with the source distance. Due to the 'coarse' source sampling the Fresnel zone is not properly sampled, which gives the alias artifacts present in the top right hand-side of the slide.

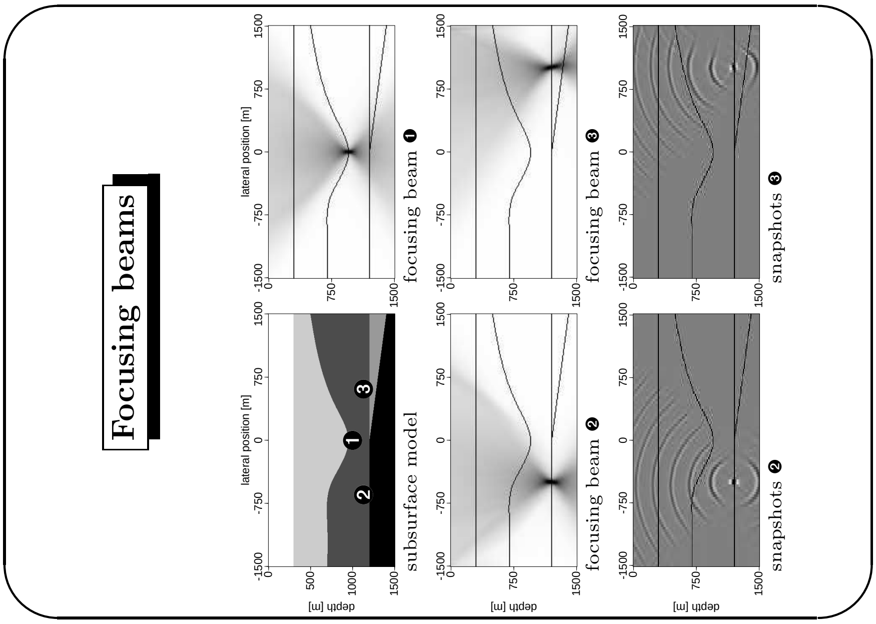
Slide 9



A simplified computation scheme for the first focusing step is shown in **slide** 9 where the difference between focusing in emission and focusing in detection is made explicitly. The focusing operator defines the traveltimes from the focus to the source and receiver positions which belongs to the input trace. The focusing blocks use this traveltime to weight the input trace. The weighting of the input trace consist in general of a time shift, a 45 degrees phase rotation and an amplitude scaling. The weighted input trace is positioned into the CFP gather at the source and/or receiver position. Other input traces which have their source or receiver position in common with the previous input trace(s) will be added to the result.

In conclusion; focusing in emission synthesizes areal sources that aim at remote illumination of single subsurface grid-points; the involved synthesis process transforms field records into CFP gathers. Hence, each CFP gather defines an areal shot record that represents the response of a focusing source. A single event in the CFP gather may be interpreted as the response of a source at the defined grid-point measured at the surface. Focusing the CFP gather gives a double focusing result. A single event in the double focusing result may be interpreted as the output of a simulated physical experiment; the response of a focusing source being measured by a focusing detector with the same focus point (confocal) or with a different focus point (bifocal). How the second focusing step is calculated will be explained in the next section.

Slide 10



Modeling the energy for a propagating wavefront of the synthesis operator through the subsurface model gives an indication how the areas in the subsurface are illuminated by this synthesis operator. From this so called focusing beam it is also possible to determine which aperture at the surface is most important for the illumination. With these illumination areas in the subsurface the distribution of the focus points in the subsurface can be determined in order to obtain an efficient and optimum illumination procedure of the subsurface. In 10 three focusing beams are shown for the syncline model introduced earlier. The beams are constructed by performing an inverse recursive depth extrapolation of the focusing operators through the model and calculating at every depth level the energy of the wavefield as function of the lateral position. Note that for the construction of the beams only the synthesis operators and a macro model are needed.

From the focusing beams it is observed that around the actual focus point most of the energy is focused in a tube like shaped area. The black center of the tube has a lateral extent which is smaller than the vertical extent. The shape of the focusing area is directly related to the resolution at the focusing point. The lateral and vertical resolution of a focus point are determined by the acquisition geometry and the subsurface model. By using this information combined with the information from the focusing beams it is possible to define at which vertical sampling density the focus points have to be chosen to illuminate the subsurface properly.

Slide 11

AVO analysis

Operator:

$$\tilde{\boldsymbol{F}}_i^-(z_m, z_0) = \tilde{\boldsymbol{I}}_i^-(z_m) \left[\mathbf{W}^+(z_m, z_0)\right]^*$$

CFP gather:

$$\tilde{\boldsymbol{P}}_i^-(z_m, z_0) = \tilde{\boldsymbol{I}}_i^-(z_m)\mathbf{R}^+(z_m)\mathbf{W}^+(z_m, z_0)\mathbf{S}(z_0)$$

If the correct focusing operator is used the time-reversed focusing operator and the focus point response have equal traveltimes. The only difference between the focus point response and its time-reversed focusing operator is in amplitude. According to equations (3) and (2), which are repeated below,

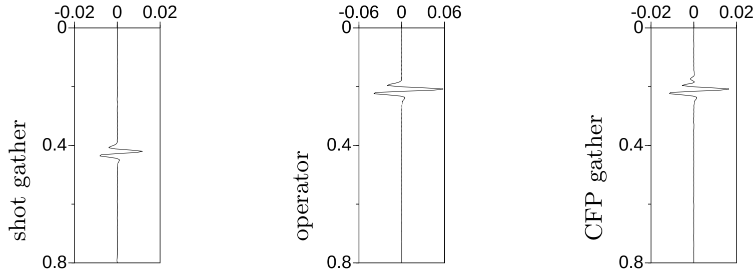
$$\tilde{\boldsymbol{F}}_i^-(z_m, z_0) = \tilde{\boldsymbol{I}}_i^-(z_m) \left[\mathbf{W}^+(z_m, z_0)\right]^* \tag{4}$$

$$\tilde{\boldsymbol{P}}_i^-(z_m, z_0) = \tilde{\boldsymbol{I}}_i^-(z_m)\mathbf{R}^+(z_m)\mathbf{W}^+(z_m, z_0)\mathbf{S}(z_0) \tag{5}$$

the difference in amplitude along the traveltime curve is given by the angle dependent reflection property at the focus point $\tilde{\boldsymbol{I}}_i^-(z_m)\mathbf{R}^+$ and the source signature $\mathbf{S}(z_0)$. Assuming that the source signature is the same for all experiments $\mathbf{S}(z_0) = \mathbf{I}S$ and that there are omni-directional receivers at the surface it is possible to derive a simple procedure which can extract the reflection information from the CFP gathers and its focusing operator. Before the amplitude analysis in a CFP gather is discussed in detail first the amplitudes for an individual trace of a shot record, a focusing operator and a CFP gather are considered.

Slide 12

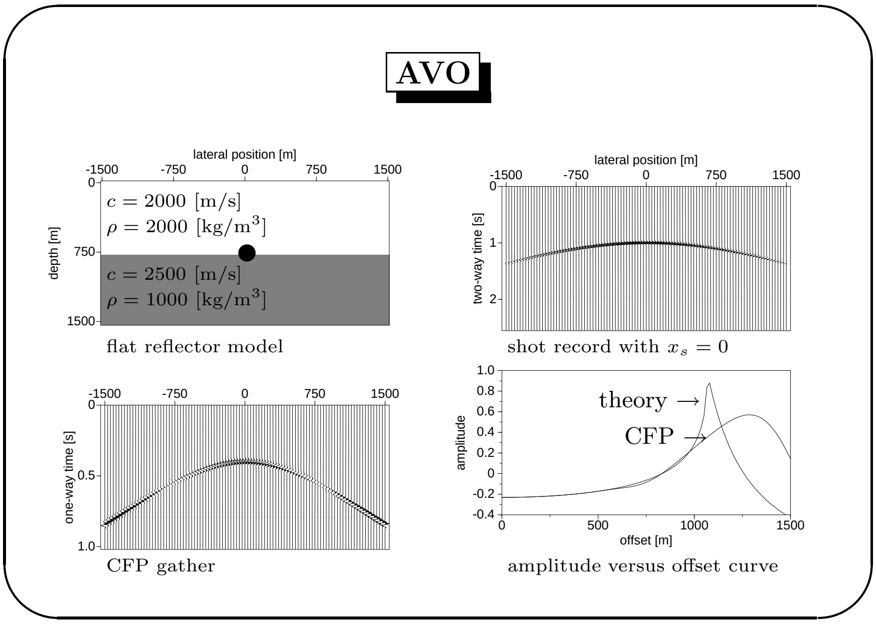
AVO



$A \simeq \frac{RS}{\sqrt{2r}}$ $A \simeq \frac{S}{\sqrt{r}}$ $A \simeq \frac{RS}{\sqrt{r}}$

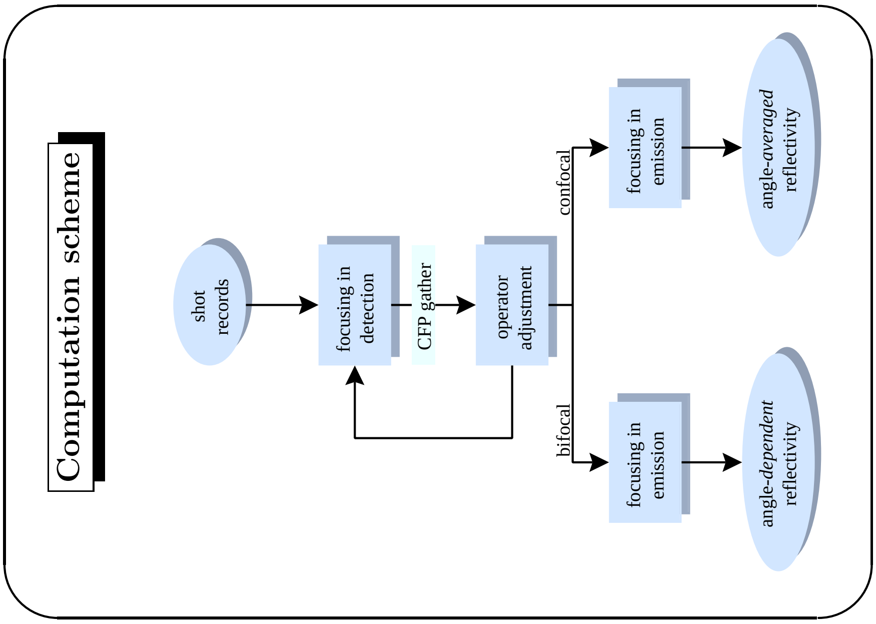
The amplitudes present in the CFP gather can be interpreted by looking at the construction of the CFP gather for the top flat layer in the syncline model shown in **slide** 12. The synthesis operator is defined for a focus point at the reflector at $x = 0$ and $z = 300$ m. The stationary phase contribution in the CFP gather of the shot record with a source position at $x = -300$ is given by the trace at $x = 300$. This stationary phase trace is shown at the left hand side. The amplitude in this trace consists of $\frac{RS}{\sqrt{2r}}$, where S represents the amplitude of the wavelet, R the reflection coefficient at 45° and r the distance between the focus point and the receiver at the surface (in the given example $r = \sqrt{2 * 300^2}$). The operator trace at $x = 300$, the middle trace, has an amplitude proportional to $\frac{1}{\sqrt{r}}$. The trace on the right hand side represents the contribution of the shot gather in the CFP gather. The amplitude in the CFP gather is proportional to $\frac{RS}{\sqrt{r}}$. Compared with the trace in the shot record there is a strong amplitude effect due to the summation in the Fresnel zone. Dividing (in a least-squares sense) the CFP gather by its synthesis operator, weighted with the wavelet (the second trace) gives the reflection coefficient for an angle of incidence of 45°. This relationship between the CFP gather and the synthesis operator can be used to determine the AVO behavior at the focus point.

Slide 13



From the foregoing discussion it can be concluded that it is possible to quantify the angle-dependent reflection property of the focus point by computing for each offset the amplitude ratio between the focus point response and the time-reversed focusing operator. This is illustrated for a flat reflector in **slide** 13; the reflector is positioned at 800 m depth and an acoustic contrast is used. The amplitude ratio shown has been computed in the time domain in a least-squares manner: for each offset the zero-lag temporal cross-correlation between the response and the focusing operator is divided by the zero-lag temporal autocorrelation of the operator. The critical reflection angle occurs at an offset of 1060 m, which is the starting point for the estimated AVO curve to deviate from the theoretical curve.

Slide 14



The flow scheme of the bifocal imaging procedure is shown in **slide** 14. After focusing in detection and optionally updating of the focusing operator the second focusing step can be carried out. The integration of velocity analysis, imaging and AVO analysis can all be combined within the same double focusing process. The analysis and processing modules are included between the two focusing steps. This does not only allow a better control on the structural solution and the reservoir characterization problem, its also offers new solutions to the well known surface-related problems, as shown in a previous paper of prof. Berkhout (?).

Slide 15

Focusing for receiver array

Correct operator:

$$\tilde{\boldsymbol{F}}_i^-(z_m, z_0) = \tilde{\boldsymbol{I}}_i^-(z_m)\left[\bar{\mathbf{W}}^+(z_m, z_0)\right]^*, \tag{6}$$

CFP gather:

$$\tilde{\boldsymbol{P}}_i^-(z_m, z_0) = \tilde{\boldsymbol{I}}_i^-(z_m)\mathbf{R}^+(z_m)\mathbf{W}^+(z_m, z_0)S_0,$$

Note that if $[\mathbf{W}^+(z_m, z_r)]^*$ represents correct propagation in the background model then the result of the first focusing step $\tilde{\boldsymbol{P}}_i^-(z_m, z_s)$ is in traveltime equal to the time reversed focusing operator $\tilde{\boldsymbol{F}}_j^+(z_s, z_m)$. To investigate the influence of erroneous operators on the CFP gather our only interest is in traveltimes. Therefore the following assumptions are made; the source function is assumed to be independent of the source position, $\mathbf{S}(z_s) = \mathbf{I}S_0$. Note that if the sources and receiver occupy the same positions in space (indicated with z_0) the focusing operators are related by $\tilde{\boldsymbol{F}}_i^+(z_0, z_m) = \left[\tilde{\boldsymbol{F}}_i^-(z_m, z_0)\right]^T$. With these assumptions the equation reduce to the equations shown on the slide. A distinction is made between propagation in the macro model $\bar{\mathbf{W}}^+(z_m, z_0)$ and propagation in the true model $\mathbf{W}^+(z_m, z_s)$. Equation (6), the focusing operator, and equation (**??**), the CFP gather, show the 'principle of equal traveltime' in its most simple form, if the propagation matrices are equal, $\bar{\mathbf{W}}^+(z_m, z_r) = \mathbf{W}^+(z_m, z_s)$, then the time-reversed focusing operator is in traveltime equal with the CFP response. Suppose that the macro model is not correct and that the true propagation matrix can be represented by

Slide 16

Erroneous operator

Model:

$$\mathbf{W}^{-}(z_0, z_m) = \bar{\mathbf{W}}^{-}(z_0, z_m)\Delta\mathbf{W}(z_m),$$

Operator:

$$\left[\tilde{\boldsymbol{F}}_i^{-}(z_m, z_0)\right]^* = \tilde{\boldsymbol{I}}_i^{-}(z_m)\left[\Delta\mathbf{W}(z_m)\right]^* \mathbf{W}^{+}(z_m, z_0),$$

CFP gather:

$$\tilde{\boldsymbol{P}}_i^{-}(z_m, z_0) = \tilde{\boldsymbol{I}}_i^{-}(z_m)\Delta\mathbf{W}(z_m)\mathbf{R}^{+}(z_m)\mathbf{W}^{+}(z_m, z_0)S_0,$$
$$\tilde{\boldsymbol{P}}_i^{-}(z_m, z_0) = \tilde{\boldsymbol{R}}_i^{+}(z_m)\Delta\mathbf{W}(z_m)\mathbf{W}^{+}(z_m, z_0)S_0,$$

The error in the model is introdced as

$$\mathbf{W}^{-}(z_0, z_m) = \bar{\mathbf{W}}^{-}(z_0, z_m)\Delta\mathbf{W}(z_m), \tag{7}$$

where $\Delta\mathbf{W}(z_m)$ is the propagation effect of the macro model error. Then the time reversed focusing operator is defined as

$$\left[\tilde{\boldsymbol{F}}_i^{-}(z_m, z_0)\right]^* = \tilde{\boldsymbol{I}}_i^{-}(z_m)\left[\Delta\mathbf{W}(z_m)\right]^* \mathbf{W}^{+}(z_m, z_0), \tag{8}$$

where $\left[\bar{\mathbf{W}}^{+}(z_m, z_0)\right]^*$ in equation is replaced by $\Delta\mathbf{W}(z_m)\left[\mathbf{W}^{+}(z_m, z_0)\right]^*$, and the CFP response as

$$\tilde{\boldsymbol{P}}_i^{-}(z_m, z_0) = \tilde{\boldsymbol{I}}_i^{-}(z_m)\Delta\mathbf{W}(z_m)\mathbf{R}^{+}(z_m)\mathbf{W}^{+}(z_m, z_0)S_0. \tag{9}$$

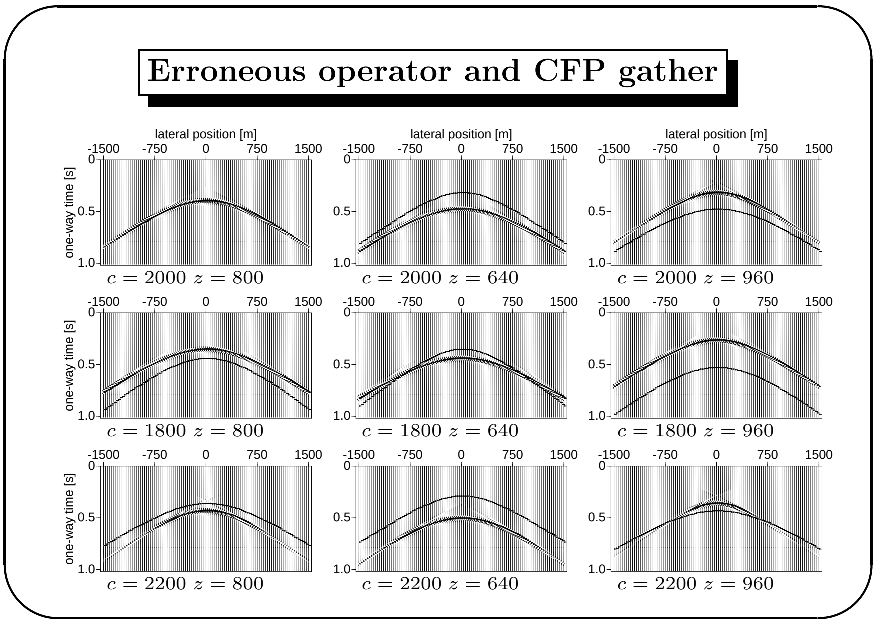
Assuming that $\Delta\mathbf{W}(z_m)$ and $\mathbf{R}^{+}(z_m)$ are Toeplitz matrices, which means that the reflection and propagation properties don't change with respect to the lateral position in the area of interest, they can be interchanged, which results in

$$\tilde{\boldsymbol{P}}_i^{-}(z_m, z_0) = \tilde{\boldsymbol{R}}_i^{+}(z_m)\Delta\mathbf{W}(z_m)\mathbf{W}^{+}(z_m, z_0)S_0, \tag{10}$$

where $\tilde{\boldsymbol{R}}_i^{+}(z_m)$ is a row vector representing the i^th row of the reflection matrix $\mathbf{R}^{+}(z_m)$.

From equations (8) and (10) it follows that the propagation errors $\Delta\mathbf{W}(z_m)$ in the time reversed focusing operator and the related focus point response are equal in magnitude and opposite in phase. This is not a surprise because the propagation time which is subtracted too much in the first focusing step should be too little in the CFP response since the total propagation time of the data is fixed. After the construction of the CFP gather the arrival times of the erroneous focusing operator can be compared with the response in the CFP gather. As indicated by equation (8) and (10) the principle of equal traveltime is no longer valid for the erroneous operator and the related focus point response. The macro model and/or the operator must be updated to obtain a correct operator which obeys the principle of equal traveltime.

Slide 17



Using an erroneous focusing operator will give a different construction of the CFP gather. This is shown in **slide** 17 for several errors in the focusing operator for shot records from a 1-dimensional medium with a fixed acquisition spread ranging from -1500 to 1500 m. The reflector in the 1-dimensional medium is positioned at 800 m depth and the velocity of the first layer is 2000 m/s. The introduced errors in the focusing operators give CFP gathers which are not time coincident with the times defined by the operator. Due to the rror in the operator the Fresnel zone is shifted in time and space, with respect to the correct position. This shift in the Fresnel zone disturbs the integration result rigorously when the Fresnel zone is shifted outside the aperture range. Note that the focusing operators with a positive velocity error can be interpreted as a high angle filter which cuts out the higher angles in the CFP gather response as shown, or shifts the Fresnel zone easily outside the aperture range.

Slide 18

Operator updating

traveltime updating:

$$\bar{T}_s^1(x_r) = \bar{T}_s^0(x_r) + T_c(x_r)$$

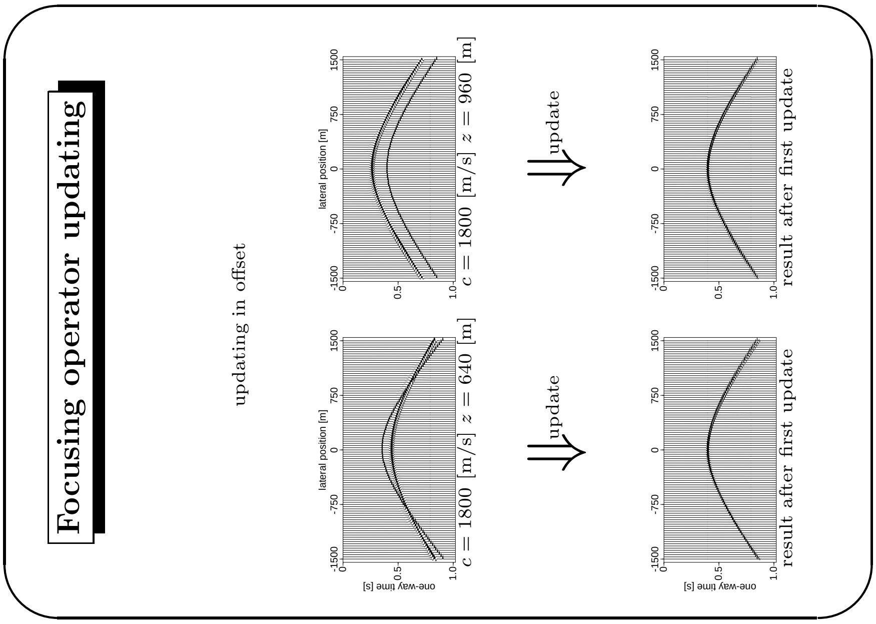
traveltime correction:

$$T_c(x_r) = \frac{T_{cfp}(x_r) - \bar{T}_s^0(x_r)}{2}$$

An simple attempt is made to detec,t from the erroneous CFP gather, an update formula for the focusing operator.

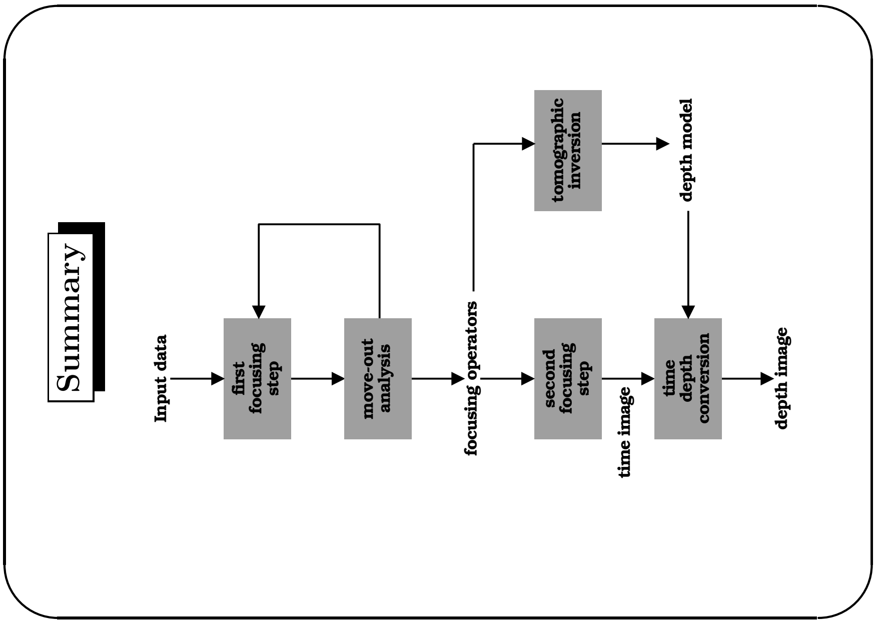
$T_c(x_r)$ is the used correction and represents the traveltime in between the operator and the focus point response at every offset. This operator update approach gives the correct operator (using hyperbolic operators) for $x_r = 0$, but for larger CFP offsets the linear update is less accurate. The accuracy at the larger offsets can be improved by making use of a second updating step.

Slide 19



In **slide** 17 two examples from the previous slides are used to demonstrate the convergence of the proposed updating scheme. The examples contain a combined depth/velocity error in the focusing operator for a model with one flat layer. From the results it is observed that although the initial macro model and the CFP gather are different in both examples they converge to the same answer within one iteration. This updating of the CFP gather can be carried out in an automatic way and is part of the research carried out by Scott Morton, who works in the same group as I'm working.

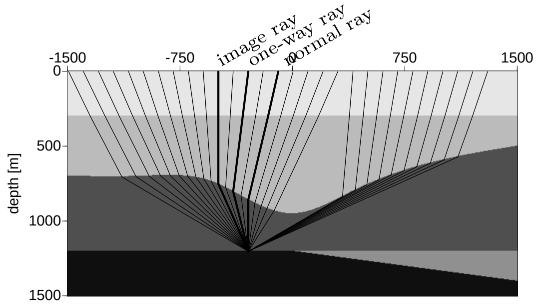
Slide 20



Using all the new analysis tools which are introduced by the CFP technology we arrive at the following computation scheme for seismic data. After the first focusing step the operator is compared with its CFP response (which hopefully can be done automatic) and if they do not fit within a certain error the operator is updated and the CFP gather is calculated again. The operator updating can be done with the simple offset interpolation as I have shown or with more advanced techniques. If all the operators in a certain area of interest are updated we have generated a collection of operators. These operators can be used in different ways; 1) Using the operators try to estimate the macro model with one global inversion which uses all the operators (next presentation by Rob Hegge) 2) Use the oparators to build an image in time (the remaining of this presentation). The imaging step in the CFP technology consists of a second focusing step.

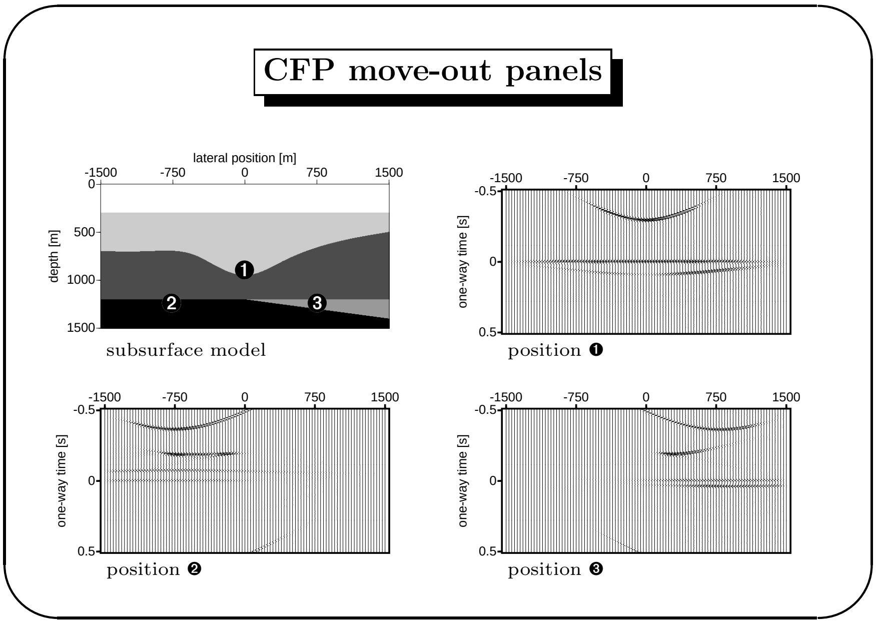
Slide 21

CFP Imaging



In migration techniques there are several rays defined which play an important role in time and depth migration Hubral defines the image ray as "the ray associated with the minimum traveltime from a subsurface point to the surface". The normal ray is associated with the minimum traveltime from a coincident source/receiver pair to any particular interface. By definition the normal ray is perpendicular to the target interface and the image ray is perpendicular to the surface. Depth migration of data has the effect of moving points along their image ray to their correct position. Time migration of data has the effect of moving points laterally to their minimum time positions (the normal ray), rather than their 'true' time positions. By the CFP technology a new ray is introduced: the one-way image ray. The one-way image ray is associated with the traveltime of a focus point in the subsurface to a receiver at the same lateral position as the focus point at the surface. In the slide the one-way image ray is displayed together with the image ray and the normal ray. Note that if the image ray is used for imaging a lateral shift of the data points is needed to position the reflector at its correct position. With the definition of the one-way image ray the result of the double focusing procedure for one focus point is positioned in the one-way image gather at the lateral position of the focus point at the one-way image time.

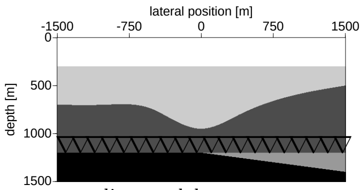
Slide 22



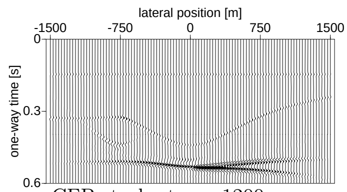
The first procedure in the second focusing step is correcting the CFP gathers with the times of the focusing operator. The resulting gather is called the CFP move-out panel. If the operator is correct the event where the operator is focusing for should align at $t = 0$. This is observed in the pictures shown on the slide. Note that beside the event around $t = 0$ there are also other events in the CFP move-out panels. Stacking the traces of the move-out panels gives an image trace which represents the correct image for $t = 0$.

Slide 23

CFP stack



syncline model

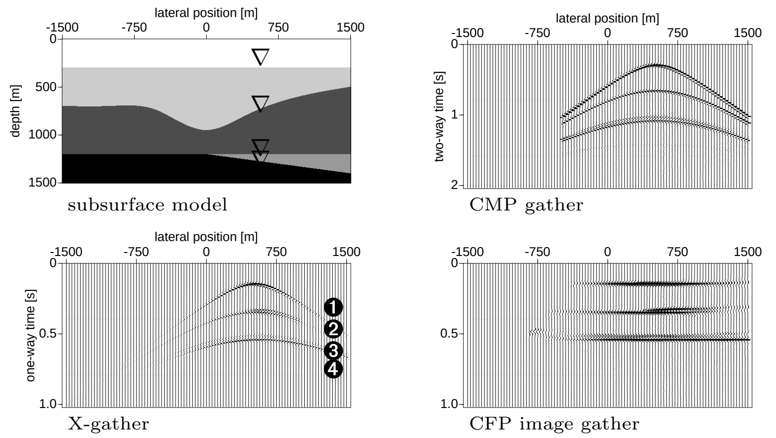


CFP-stack at $z = 1200$ m

Repeating this stacking for focal point positions at different lateral positions gives the so called CFP-stack. In the CFP-stack the event at the image time of the defined focus point is equal to the double focusing result. Around the one-way image time the CFP-stack will give a good representation of the area surrounding the focusing point. The CFP-stack can be used to get quick an idea of the main reflectors in the data, but it can also be used to investigate a particular reflector or depth level.

Slide 24

CFP image gather



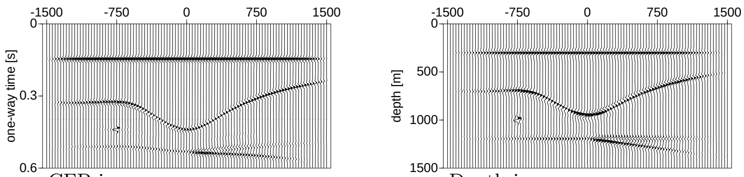
The CFP gather gives the response from one focus point in the subsurface. The most important information in the CFP gather is therefore concentrated around the focus point response. The CFP image gather is constructed by selecting samples from the CFP gather around a time window, defined by the focusing operator, and placing these selected time samples into one gather. This procedure is repeated for different CFP gathers constructed with focusing operators defined at the same lateral position, but at different depths (or one-way times). In the CFP image gather the samples from the CFP gather are converted to one-way time with respect to the one-way image time of the focusing operator. For the move-out correction in the CFP image gather at every one-way time sample an operator is needed. These operators are obtained by a linear interpolation (in offset or in ray-parameter p) of the operators used in the first focusing step. With these operators the CFP image gather can be move-out corrected and stacked to get the image trace. This move-out corrected multi focus gather represents the CFP image gather in one-way image time. If the move-out correction is not applied the gather is called an X-gather (X from exploding).

slide 24 shows a CFP image gather, positioned at $x = 500$ m, for the syncline model. The image gather is constructed from 4 CFP gathers with their focus points defined at the boundaries of the model. Event ❶ originates from the flat top reflector, ❷ from the right flank of the syncline (note the finite aperture artifact due to the dip at the flank), ❸ originates from the deep flat reflector (note the negative reflection coefficient) and ❹ from the wedge in the right part of the model. The X-gather (c) can also be used as an alternative for the CMP gather which is also shown.

Building the CFP image gather, by making use of traveltime operators, introduces a stretch of the wavelet due to the convergence of the operator times at the higher offsets. This stretched part at the higher offsets can be removed from the CFP image gather by setting a stretch parameter.

Slide 25

CFP image



CFP image

Depth image

Imaging results for the double focus procedure using synthesis operators based on a homogeneous model; with three levels of focus points and the correct model. The right hand side result shows the pre-stack depth migration result using the correct macro model.

Slide 26

Concluding remarks

- CFP gathers are very well suited for velocity analysis.
- The focusing operator can be updated without using a velocity model
- The collection of updated focusing operators can be used to estimate the macro model (next presentation).
- A postscript version of the slides is available from http://reality.sgi.com/jant/Presentations.

CRAY RESEARCH
A Silicon Graphics Company