# G032 SEISMIC INTERFEROMETRY: RECONSTRUCTION OF THE ELASTODYNAMIC REFLECTION RESPONSE

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# Introduction

In [1] it was shown that for 1-D acoustic media the reflection response can be synthesized by taking the autocorrelation of the transmission response measured at the free surface. This method was coined "Acoustic Daylight Imaging". In [2] this was mathematically generalized for a 3-D inhomogeneous medium, both acoustic and elastic. In this paper, the relation between the reflection and the transmission responses were derived making use of an one-way reciprocity theorem of the correlation type. Numerical acoustical modelling confirmed these relations for the acoustical situation and we investigated the quality of the synthesized reflection response depending on the source distribution and time duration ([3], [4]).

Here, we show modelling results for elastic media using a relation between the reflection and the transmission responses at the free surface derived from a two-way reciprocity theorem of the correlation type.

## Theory

Consider an elastic inhomogeneous anisotropic lossless medium bounded by a free surface. In this medium a volume  $\mathcal{D}$  is taken such that part of its boundary  $\partial \mathcal{D}$  consists of a piece of the free surface ( $\partial \mathcal{D}_0$ ) and of an arbitrary shaped surface inside the medium ( $\partial \mathcal{D}_1$ ). The following relation between Green's functions can be written in the frequency domain ([5]):

$$2\Re \left\{ \hat{G}_{p,q}^{v,t} \left( \boldsymbol{x}_{A}, \boldsymbol{x}_{B}, \omega \right) \right\} = -\int_{\partial \mathcal{D}_{1}} \left[ \left\{ \hat{G}_{p,i}^{v,f} \left( \boldsymbol{x}_{A}, \boldsymbol{x}, \omega \right) \right\}^{*} \hat{G}_{q,i}^{v,h} \left( \boldsymbol{x}_{B}, \boldsymbol{x}, \omega \right) + \left\{ \hat{G}_{p,i}^{v,h} \left( \boldsymbol{x}_{A}, \boldsymbol{x}, \omega \right) \right\}^{*} \hat{G}_{q,i}^{v,f} \left( \boldsymbol{x}_{B}, \boldsymbol{x}, \omega \right) \right] d^{2}\boldsymbol{x}.$$
(1)

In equation (1) the first superscript (v - velocity) stands for the observed quantity and the second (t - traction, f - force or h - deformation rate) - for the source quantity. The first subscript (p, q) describes the component of the observed quantity, while the second (q, i) - the component of the source quantity. On the left-hand side of equation (1) stands the real part of the Green's function, which represents the particle velocity in the  $x_p$  direction measured at point  $x_A$  from a traction source at point  $x_B$  in the  $x_q$  direction. The points  $x_A$  and  $x_B$  are situated on  $\partial D_0$ . The right-hand side of the above relation represent the cross-correlations between the particle velocities measured in the  $x_p$  and  $x_q$  directions at the points  $x_A$  and  $x_B$  due to subsurface sources at points x along the surface  $\partial D_1$ .

If the medium outside  $\partial D_1$  is homogeneous and contains no sources, then it can be shown that equation (1) can be approximated by

$$2\Re\left\{\hat{G}_{p,q}^{v,t}\left(\boldsymbol{x}_{A},\boldsymbol{x}_{B},\omega\right)\right\}\approx-\int_{\partial\mathcal{D}_{1}}\left\{\hat{G}_{p,k}^{v,\phi}\left(\boldsymbol{x}_{A},\boldsymbol{x},\omega\right)\right\}^{*}\hat{G}_{q,k}^{v,\phi}\left(\boldsymbol{x}_{B},\boldsymbol{x},\omega\right)d^{2}\boldsymbol{x}.$$
(2)

In the right-hand side of relation (2) the Green's functions again stand for measured particle velocities at the free surface due to sources along  $\partial D_1$ . The superscript  $\phi$  stands for quasi P-wave sources (when k = 1)

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and quasi S-wave sources (when k = 2, 3). These quasi P- and S-sources are obtained by applying fluxnormalized decomposition at the source level. The accuracy of relation (2) depends on the curvature of the boundary  $\partial D_1$ . When  $\partial D_1$  is planar the only approximation is that the evanescent fields are not taken into account.

# **Modelling results**

Fig. 1 shows a two-layer model used to reconstruct the elastodynamic reflection response according to Eq. 2. The upper layer has a P-wave velocity of 2500 m/s, S-wave velocity of 2000 m/s and density of 1800 kg/m<sup>3</sup>. The seismic parameters of the lower layer are: P-wave velocity of 3300 m/s, S-wave velocity of 2500 m/s and density of 4000 kg/m<sup>3</sup>.



Figure 1: Elastic model with free surface. There are impulsive P- and S- sources at depth level  $x_3$ =800 m starting at  $x_1$ =2100 m until  $x_1$ =5700 m every 15 m. The receivers are situated at the free surface from  $x_1$ =2100 m until  $x_1$ =5700 m every 15 m.

The receiver spread is placed at the free surface ( $x_3=0$  m) between horizontal coordinates  $x_1=2100$  m until  $x_1=5700$  m every 15 m. The source positions were chosen at depth coordinate  $x_3=800$  m and with horizontal positions every 15 m between the points  $x_1=2100$  m and  $x_1=5700$  m.

The wave propagation simulations were performed using a finite element code ([6]). At each source position separate shots were simulated with a P-wave and a S-wave source. Fig. 2 depicts example transmission responses showing the vertical particle velocity component from P-wave source at subsurface point with coordinate x=(3900,800) (a) and the vertical particle velocity component from a S-wave source at the same subsurface point (b).

To reconstruct the vertical particle velocity component reflection response from a traction source in the vertical direction at the surface according to Eq. (2), we follow the procedure below. The trace at horizontal position  $x_1$ =3900 m in the P-wave transmission panel from Fig. 2 (a) is extracted and cross-correlated with all the traces in the same panel. The same is done for vertical particle velocity components from the recorded transmission panels due to all other P-wave sources in the subsurface. The results from the cross-correlation are then summed. The end result, after muting the negative times, is shown in Fig. 3 (a).

The same procedure is repeated for the vertical particle velocity component transmission panels due to S-wave subsurface sources (like the one in Fig. 2 (b)) and the result is shown in Fig. 3 (b).

As a last step, the two panels from Fig. 3 are summed together to produce the final simulated reflection response of the layered medium from Fig. 1. The result is shown in Fig. 4 (a). Comparison of the simulated



Figure 2: (a) Recorded vertical particle velocity transmission shot panel from a P-wave source at subsurface point x with coordinates  $x_1=3900$  m and  $x_3=800$  m. (b) Recorded vertical particle velocity transmission shot panel from a S-wave source at subsurface point x with coordinates  $x_1=3900$  m and  $x_3=800$  m.



Figure 3: (a) Result from cross-correlating the vertical particle velocity component transmission data due to P-wave subsurface sources and summing along the source positions in accordance with Eq. (2). (b) Result from cross-correlating the vertical particle velocity component transmission data due to S-wave subsurface sources and summing along the source positions in accordance with Eq. (2).

reflection response in (a) with the directly modelled reflection response in (b) confirms the validity of relation (2). Note, that the directly modelled reflection response in Fig. 4 (b) is shown after filtering out the direct and the surface waves, while the simulated reflection response in Fig. 4 (a) is shown as it was produced (no surface wave removal was applied). As explained in [5], when there are no subsurface sources close to the free surface only the reflection response will be reconstructed after correlation.



Figure 4: (a) Simulated reflection response for the model in Fig. 1 resulting from the summation of the panels in Fig. 3 (a) and (b) according Eq. (2). (b) Directly modelled reflection response for the model in Fig. 1 after filtering out the direct and the surface waves.

## Conclusions

Here we showed that the elastodynamic reflection response of a 3-D inhomogeneous anisotropic lossles medium can be reconstructed from its transmission response. This is done, in accordance with Eq. 2, by cross-correlating separately the recorded transmissions from P- and S-wave sources in the subsurface. When there are no subsurface sources close to the free surface, the reconstructed reflection response does not contain surface waves.

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