A041
From Reflection to Transmission Data

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SUMMARY

From the reciprocity theorem of the correlation type an implicit relation between seismic reflection and transmission data has previously been derived. In this paper this relation is used to estimate the transmission coda, at a certain depth, from reflection data measured at the surface. The transmission response is represented by a generalized propagator, consisting of a primary propagator and a coda operator. Using this representation, 1D approximations, and an eigenvalue decomposition on the correlation of the reflection response, it is possible, to solve the implicit relation for the coda operator. The calculated coda response may be used in seismic reflection imaging to obtain an image in which the internal multiple scattering effects are suppressed. A simple example for the estimation of the transmission coda illustrates the discussed procedure.

Introduction

The one-way reciprocity relation of the correlation type in the frequency domain,

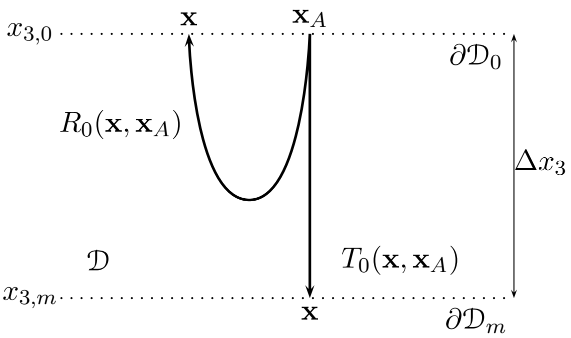
$$\int_{\partial\mathcal{D}_0} \{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\} d^2\mathbf{x}_H = \int_{\partial\mathcal{D}_m} \{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\} d^2\mathbf{x}_H \tag{1}$$

is the starting point to derive a relation between reflection and transmission data. The medium parameters in both states A and B are assumed to be identical, lossless and 3-D inhomogeneous, and the domain $\mathcal{D}$ is source free. $\partial\mathcal{D}_0$ and $\partial\mathcal{D}_m$ are two horizontal boundaries at $x_{3,0}$ and $x_{3,m}$, respectively, enclosing the domain $\mathcal{D}$ (see figure (1)). $\mathbf{x}_H$ denotes the horizontal coordinates (x_1, x_2) and * denotes complex conjugation. Both states, and the related flux-normalized up- and down-going fields P^- and P^+ respectively, are chosen as shown in the table in figure (1).

Substituting the two states of the table into equation (1) results in (Wapenaar et al. (2004)):

$$\delta(\mathbf{x}_{H,A} - \mathbf{x}_{H,B}) - \int_{\partial\mathcal{D}_0} R_0^*(\omega, \mathbf{x}, \mathbf{x}_A) R_0(\omega, \mathbf{x}, \mathbf{x}_B) d^2\mathbf{x}_H = \int_{\partial\mathcal{D}_m} T_0^*(\omega, \mathbf{x}, \mathbf{x}_A) T_0(\omega, \mathbf{x}, \mathbf{x}_B) d^2\mathbf{x}_H. \tag{2}$$

Here $R_0(\mathbf{x}, \mathbf{x}_A)$ is the reflection response of the inhomogeneous medium in $\mathcal{D}$, including all internal multiples, for a source at $\mathbf{x}_A = (\mathbf{x}_{H,A}, x_{3,0})$ and a receiver at $\mathbf{x} \in \partial\mathcal{D}_0$. $T_0(\mathbf{x}, \mathbf{x}_A)$ is the transmission response of the medium in $\mathcal{D}$ with receivers at $\mathbf{x} \in \partial\mathcal{D}_m$. The subscript $_0$ in R_0 and T_0 denotes that free surface multiples are not included. There is no unique way to solve the full transmission response from equation (2) (Herman, 1992). However, by using a suitable approximation of the transmission operator there is a way to solve the transmission coda.



Surface $\partial\mathcal{D}_0$		
Field	**State A**	**State B**
P^+	$\delta(\mathbf{x}_H - \mathbf{x}_{H,A}) s_A(\omega)$	$\delta(\mathbf{x}_H - \mathbf{x}_{H,B}) s_B(\omega)$
P^-	$R_0(\mathbf{x}, \mathbf{x}_A, \omega) s_A(\omega)$	$R_0(\mathbf{x}, \mathbf{x}_B, \omega) s_B(\omega)$
Surface $\partial\mathcal{D}_m$		
P^+	$T_0(\mathbf{x}, \mathbf{x}_A, \omega) s_A(\omega)$	$T_0(\mathbf{x}, \mathbf{x}_B, \omega) s_B(\omega)$
P^-	0	0

Figure 1: *At $\mathbf{x}_A$ (state A, state B not shown) just above $\partial\mathcal{D}_0$ there is a source for downgoing waves. The half-spaces above $\partial\mathcal{D}_0$ and below $\partial\mathcal{D}_m$ are homogeneous. The half-space below $\partial\mathcal{D}_m$ is source free. The wave fields of both states are shown in the table in the text.*

1 Forward model of Transmission response

Rewriting equation (2) in matrix form, and leaving out the dependency on ω, results in

$$\mathbf{T}_0^H(x_{3,m}, x_{3,0}) \mathbf{T}_0(x_{3,m}, x_{3,0}) = \mathbf{I} - \mathbf{R}_0^H(x_{3,0}) \mathbf{R}_0(x_{3,0}). \tag{3}$$

A column of matrix $\mathbf{R}_0(x_{3,0})$ contains the discretized version of $R_0(\mathbf{x}, \mathbf{x}_A, \omega)$ for a fixed source position $\mathbf{x}_A$ and a range of receiver positions $\mathbf{x}$ at $x_{3,0}$. $\mathbf{I}$ is the identity matrix. The superscript ^{H} denotes transposition and complex conjugation.

Equation (3) can also be interpreted as a generalized expression for conservation of energy. The 'energy' which enters and leaves the two depth levels: the transmitted data through the medium $\mathbf{T}_0$ and the reflected data $\mathbf{R}_0$, is equal to the 'energy' going into the medium: $\mathbf{I}$. Note that equation (3) is a generalization, since the conservation of energy is expressed by the diagonal terms only. This relation is illustrated in Figure 2 for a simple 1D medium, and a more complex medium containing a syncline shaped layer. Each panel represents a column of the considered matrix, inverse Fourier transformed to the time domain.

To construct the correlation panel, for the syncline model shown in Figure 3a, the middle shot record is selected and correlated with all the other shot records. Thus each shot record contributes to one trace in the

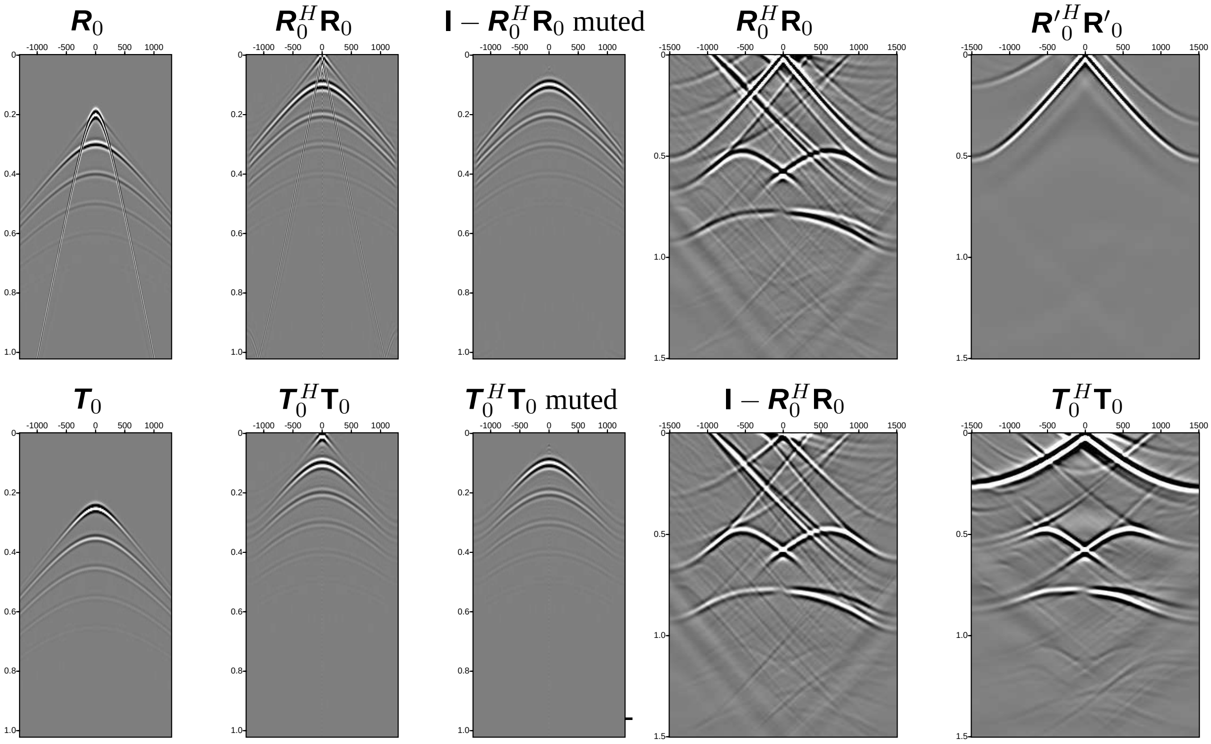


Figure 2: *The generalized conservation of energy expressed in equation (3) shown for a single, high contrast, layer (left 6 pictures) and for a syncline shaped medium (right 4 pictures). The correlated transmission response is equal to the direct contribution (**I**) minus the correlated reflection response.*

correlation panels. The bottom pictures show again that both correlated panels are similar to each other. The panel labeled $\mathbf{R}'^H_0\mathbf{R}_0$ shows the correlation of the data with the first reflector only $\mathbf{R}'_0$. This panel represents $\mathbf{I}$ where the effect of the limited aperture is included. The differences at the edges of the correlated reflection and transmission panels are caused by the differences in illumination between transmission and reflection data. The reflection data contains more angle information of the subsurface than the transmission data, if the acquisition set-up is kept the same.

The transmission response is written as a generalized propagator $\mathbf{W}_g$, which consists of two parts

$$\mathbf{T}_0(x_{3,m}, x_{3,0}) = \mathbf{W}_g(x_{3,m}, x_{3,0}) = \mathbf{W}_p(x_{3,m}, x_{3,0})\mathbf{C}(\Delta z) \tag{4}$$

with $\mathbf{W}_p$ the primary propagator, $\mathbf{C}$ the coda operator and $\Delta z = x_{3,m} - x_{3,0}$. The coda operator is a causal function describing the distortion of $\mathbf{W}_g$ relative to $\mathbf{W}_p$, and accounts for the effect of inhomogeneities between two depth levels. These effects include multiple reflections which can delay, shape and magnify the transmitted pulse.

Substituting equation (4) into equation (3), leaving out the dependencies on the parameters, and using $\mathbf{W}_p^H(x_{3,m}, x_{3,0})\mathbf{W}_p(x_{3,m}, x_{3,0}) \approx \mathbf{I}$, results in

$$\mathbf{T}_0^H\mathbf{T}_0 = (\mathbf{W_p C})^H\mathbf{W_p C} = \mathbf{C}^H\mathbf{C} = \mathbf{I} - \mathbf{R}_0^H\mathbf{R}_0 \tag{5}$$

Equation (5) states that the auto-correlation of the transmission coda ($\mathbf{C}^H\mathbf{C}$), can be obtained from the auto-correlation of the reflection matrix.

We have to resolve $\mathbf{C}$ from $\mathbf{C}^H\mathbf{C}$ and make therefore the assumption that:

$$\mathbf{C} = \mathbf{L}\boldsymbol{\Lambda}_c\mathbf{L}^H \tag{6}$$

where

$$\boldsymbol{\Lambda}_c = \exp\{-\mathbf{A}\} = \begin{pmatrix} e^{-\mathcal{A}(\omega,p_1,\Delta z)} & 0 & \dots & 0 \\ 0 & e^{-\mathcal{A}(\omega,p_2,\Delta z)} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & e^{-\mathcal{A}(\omega,p_N,\Delta z)} \end{pmatrix} \tag{7}$$

where $\mathbf{A}$ is a diagonal matrix depending on ω and parameter p. The elements $\mathcal{A}(\omega, p_i, \Delta z)$ being the temporal Fourier transform of causal functions, where the parameter p corresponds to the ray-parameter. For horizontally layered media O'Doherty and Anstey (1971) found that the coda operator $C(\omega)$ is related to the power spectrum of the reflection coefficient

$$C(\omega, p, \Delta z) = \exp(-\mathcal{A}(\omega, p, \Delta z), \tag{8}$$

and in that case the assumptions made in equation (6) are valid. For plane waves in horizontally layered media $\mathbf{C}$ is a circulant matrix, which has the property that its Fourier transform is equal to its eigenvalues:

$$\mathbf{\Lambda}_c = \mathbf{F}\mathbf{C}\mathbf{F}^H \tag{9}$$

Note that for this situation $\mathbf{L} = \mathbf{F}^H$.

2 Construction of $\mathbf{C}$ from $\mathbf{R}_0$.

Using the assumption on the coda operator $\mathbf{C}$ the correlation of coda matrix is given by

$$\mathbf{C}^H\mathbf{C} = \mathbf{L}\mathbf{\Lambda}_c^H\mathbf{\Lambda}_c\mathbf{L}^H \quad \text{where,} \tag{10}$$

$$\mathbf{\Lambda}_c^H\mathbf{\Lambda}_c = \exp\{-2\mathcal{R}\{\mathbf{A}\}\} \tag{11}$$

The $\mathbf{A}$ operator can be retrieved from its real parts $\mathcal{R}\{\mathcal{A}(\omega, p_i)\}$,because we assumed that the elements $\mathcal{A}(\omega, p_i)$ of $\mathbf{A}$ are the Fourier transforms of causal filters in the time domain. For plane waves in 1D media this is exact, and has been shown by Wapenaar et al. (2002). Note that before the temporal Fourier transforms yields the causal filters, the diagonal elements must be scaled from wavenumber k_x to ray-parameter p with $\frac{1}{\omega}$ $(k_x = p\omega)$.

Going back to the relation between the coda and the reflection data, given by equation (5), we can now lay-out the computational procedure. First the measured data at surface $\partial\mathcal{D}_0$ is used to calculate the cross correlation of the reflection data $\mathbf{R}_0^H\mathbf{R}_0$ which relates to:

$$\mathbf{C}^H\mathbf{C} = \mathbf{I} - \mathbf{L}\mathbf{\Lambda}_r\mathbf{L}^H \tag{12a}$$

$$\mathbf{L}\mathbf{\Lambda}_c^H\mathbf{\Lambda}_c\mathbf{L}^H = \mathbf{L}[\mathbf{I} - \mathbf{\Lambda}_r]\mathbf{L}^H. \tag{12b}$$

The eigenvalues of the cross correlation matrix have now to be mapped from wavenumber (eigenvalue number) to ray-parameter p. Then the following relation gives the real part of the causal filters:

$$\exp\{-2\mathcal{R}\{\mathbf{A}\}\} = \mathbf{I} - \mathbf{\Lambda}_r \tag{13}$$

$$\mathcal{R}\{\mathbf{A}\} = -\frac{1}{2}\ln\{\mathbf{I} - \mathbf{\Lambda}_r\} \tag{14}$$

Using the Hilbert transform, the causal functions can be reconstructed from their real part, this gives $\mathcal{A}(p_i)$. Inserting these computed functions into equation (6) gives $\mathbf{C}$. With an estimation of the primary propagator the calculated coda can be used to calculate the transmission response $\mathbf{T}_0$ expressed in equation (4).

3 Examples

The columns of the cross-correlation matrix $\mathbf{R}_0^H\mathbf{R}_0$ can be interpreted as belonging to a horizontally layered medium, and eigenvalues can be calculated using the Fourier transform. The selected column is interpreted as being the first column of a Toeplitz matrix. The eigenvalues of this Toeplitz matrix are then calculated using the Fourier transform as shown in equation (9). The obtained results are mapped from wavenumber to ray-parameter. This gives the results as shown in Figure 3c,d,e and f for different columns. Note that these are the estimated Λ_c operators in the laterally variant medium of Figure 3a. At this moment we are investigating if these estimated operators can be used in equation (4) to reconstruct the transmission response.

4 Conclusions

In this paper we have shown that correlated reflection panels contain information of the transmission coda. One possible way to retrieve this information is using a local lateral invariant medium assumption. In the future we will investigate how to use these estimations in the construction of propagation operators.

Acknowledgements

We would like to thank the research school ISES for supporting this research.

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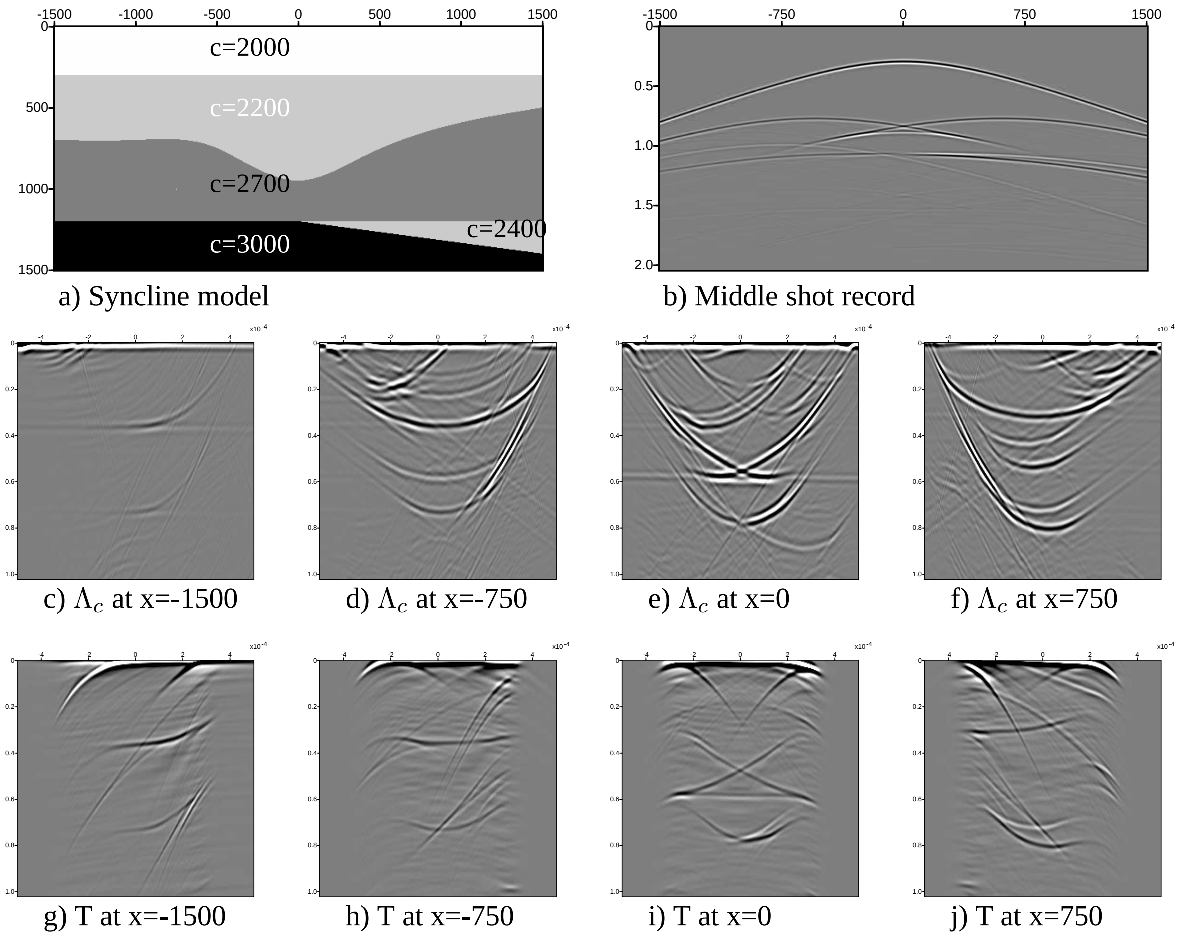

Figure 3: *Eigenvalues in $\tau - p$ for a 2D medium. The differences between the modelled coda and the calculated coda (obtained by eigenvalue decomposition, see the text) is caused by the differences in receiver aperture. For the modelled transmission response the source is positioned at $x_{3,0} = 0$ and the receivers at $x_{3,m} = 1400$ m. The modelled transmission coda has an effective receiver aperture which is different from the receiver aperture of the coda derived from the reflection data. Pictures c-f, show the computed eigenvalues of the Codas in the $\tau - p$ domain, using a local FFT transform (1D assumption). Pictures g-j show the modeled transmission response which has been inversely extrapolated ($\mathbf{W}_p^H(x_{3,m}, x_{3,0})$) to the receiver level at $x_{3,m} = 1400$ m and mapped to the $\tau - p$ domain.*