

## Controlled illumination by double focussing in the source domain

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*The CFP-method computes a virtual source in the subsurface by simulating a focal source array at the surface. The response of such a virtual source represents the CFP-gather. It is proposed to construct a distribution of virtual sources along a vertically oriented trajectory. By simulating a second focal array along this trajectory of virtual sources, a 'pencil beam' is created that illuminates the subsurface in a user-controlled way. The subsurface response of this double focal source array (DFS-array) equals a weighted sum of CFP-gathers.*

### 14.1 Introduction

In the past (Berkhout, 1997; Thorbecke, 1997) it was shown that virtual sources can be simulated in the subsurface by simulating a focal source array at the surface. Figure 14.1 shows the principle.

Using the **WRW**-model, we may write for the focal source array along the surface:

$$\mathbf{S}_j(z_0, z_m) = \mathbf{S}(z_0)\mathbf{F}_j(z_0, z_m) \quad (14.1a)$$

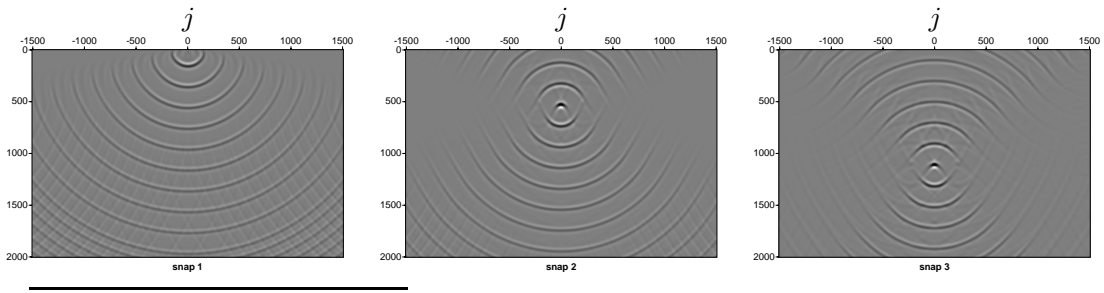
with

$$\mathbf{W}(z_m, z_0)\mathbf{S}(z_0)\mathbf{F}_j(z_0, z_m) = \mathbf{I}_j(z_m), \quad (14.1b)$$

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**Fig. 14.1** Generation of virtual sources in the subsurface by simulating focal source arrays at the surface.

$\mathbf{F}_j(z_0, z_m)$  representing the focal operator for a virtual source at  $(x_j, z_m)$  and  $\mathbf{I}_j(z_m)$  being the  $j$ 'th column of unit matrix  $\mathbf{I}(z_m)$ . The response of this focal source array is given by the CFP-gather

$$\mathbf{P}_j(z_0, z_m) = \mathbf{P}(z_0, z_0)\mathbf{F}_j(z_0, z_m), \quad (14.2)$$

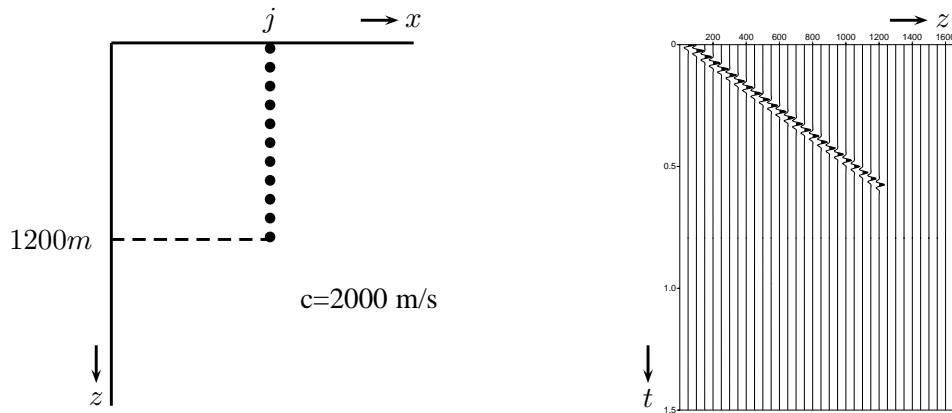
$\mathbf{P}(z_0, z_0)$  representing the data matrix.

If  $m$  ranges from  $m_1$  to  $m_2$ , a distribution of virtual sources is generated along the vertical trajectory at surface position  $j$ :

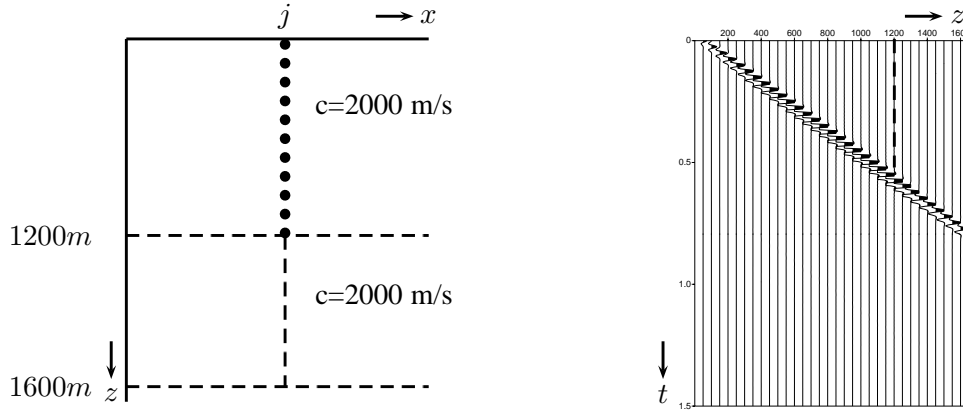
$$\mathbf{S}_j(z_m, z_m) = \mathbf{W}(z_m, z_0)\mathbf{S}(z_0)\mathbf{F}_j(z_0, z_m) \quad \text{for } m = m_1, m_1 + 1, \dots, m_2. \quad (14.3a)$$

Figure 14.2 displays the delayed source signatures at the focal points for a homogeneous medium:

$$\mathbf{S}'_j(z_m, z_m) = \mathbf{S}_j(z_m, z_m)e^{-j\omega\tau_m} \quad \text{for } m = 1, 2, \dots, 24 \quad (14.3b)$$



**Fig. 14.2** Distribution of virtual sources (left) with source signatures (right). The virtual sources are constructed by applying a focusing process to the sources at the surface, see equations (14.3a,b).



**Fig. 14.3** Source signatures of a vertical array upto 1200 m ( $m = 1, 2, \dots, 24$ ), displayed along the vertical upto 1600 m.

with  $\Delta z = 50$  m. In equation (14.3b)  $\tau_m$  is the traveltime along the trajectory between the depth levels  $z_0$  and  $z_m$ . Note that  $\tau_m$  can be found in focusing operator  $\mathbf{F}$ .

Let us introduce a second focusing step by simulating a focal source array along the vertical trajectory ( $z_{m_1}, z_{m_2}$ ):

$$\mathbf{S}_j(z_{m_2}, z_{m_1}) = \sum_{m_1}^{m_2} \mathbf{S}_j(z_m, z_m) \quad (14.4a)$$

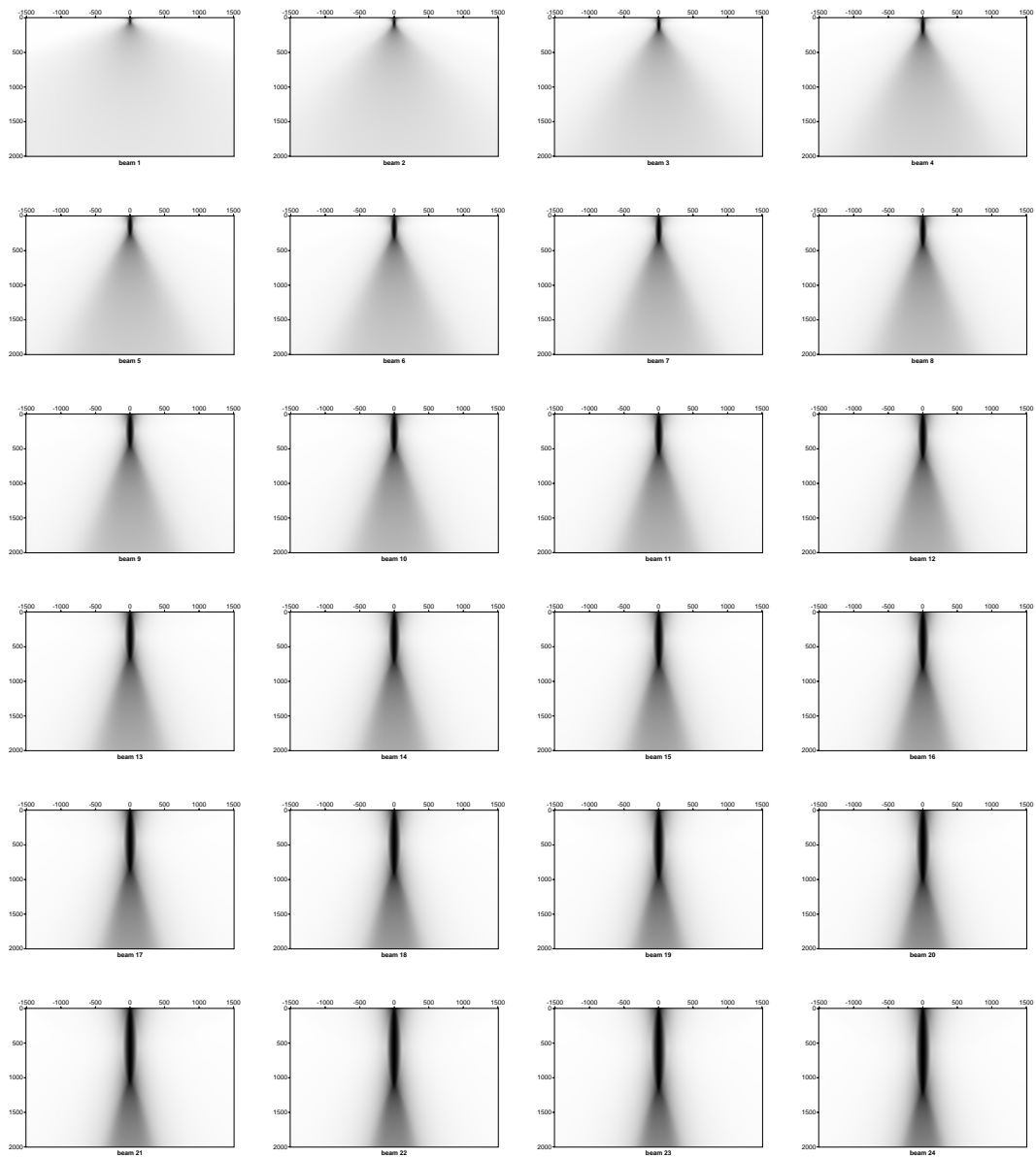
or for display purposes:

$$\mathbf{S}'_j(z_{m_2}, z_{m_1}) = \sum_{m_1}^{m_2} \mathbf{S}_j(z_m, z_m) e^{-j\omega\tau_{m_2}}. \quad (14.4b)$$

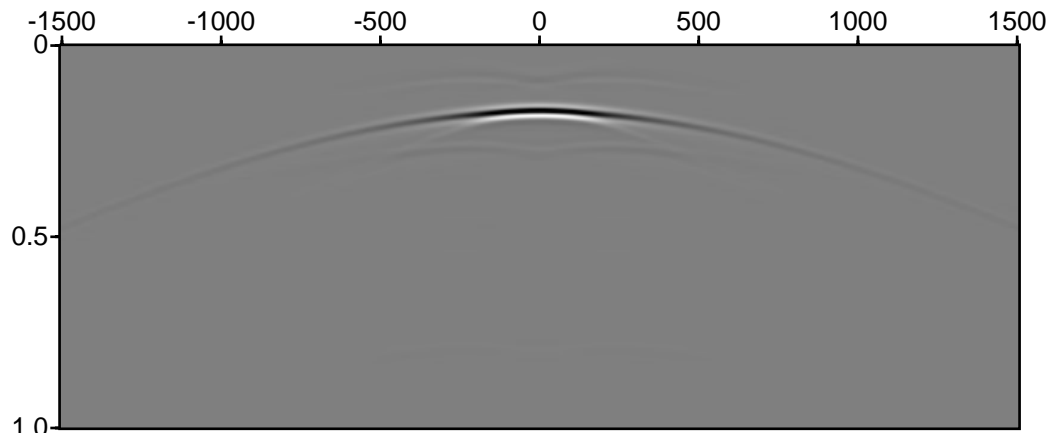
Equation (14.4b) simulates a source wavefield that is each time enhanced when it reaches the next source position in the vertical array. This is illustrated in Figure 14.3 for a homogeneous medium ( $m = 1, \dots, 24$ ). In terms of Figure 14.2, it means that the signatures in the  $z - t$  plane are added in phase and positioned at the vertical traveltime. In Figure 14.4 the beam properties of this double-focussing array are shown. Note the increasing amplitude with depth.

Figure 14.5 shows the illumination properties of the DFS-array ( $m_1 = 1, m_2 = 24$ ). Note the high amplitude at the centre of the source beam.

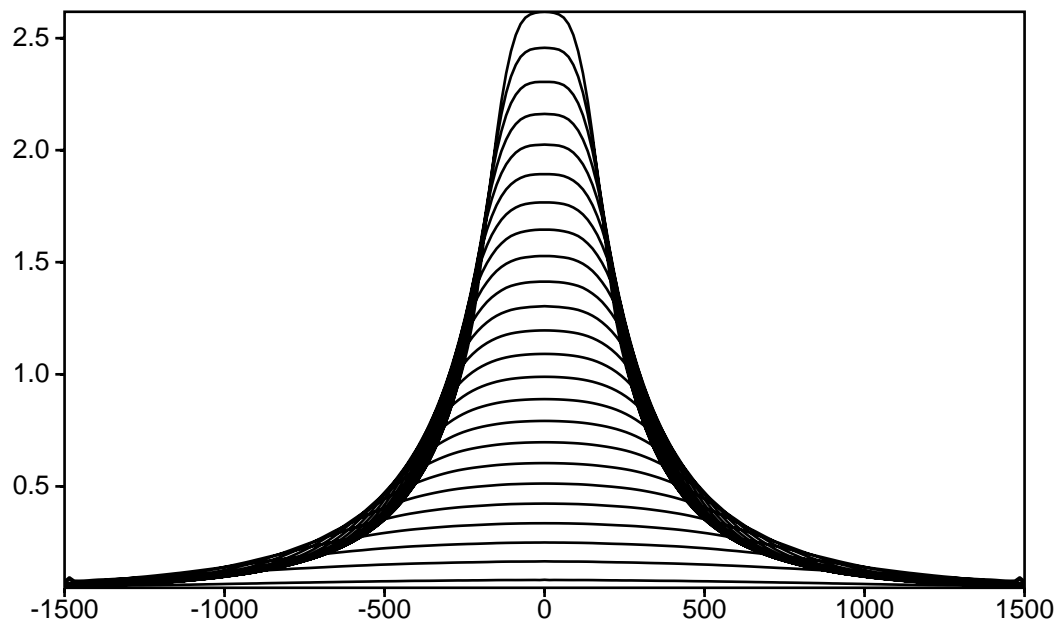
Figure 14.6a shows a simple example of a two reflector model. Note the effect of the velocity change at 800 m.



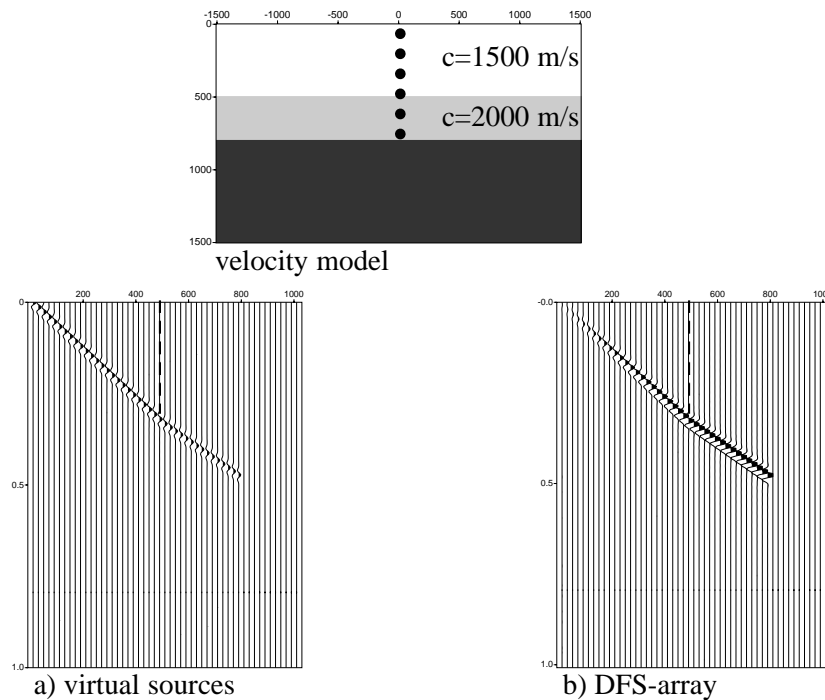
**Fig. 14.4** *Illumination by double focussing (horizontal array of physical sources, vertical array of virtual sources). The vertical array consists of 24 virtual source points with a distance of 50m [50-1200]m. Starting from  $z = 0$  every 50m the wavefield of a virtual source is added in phase to the total wavefield. Note that the beams are displayed with different maximum values for each picture.*



wavefield at  $z=1600$



**Fig. 14.5** Source wavefield, obtained by double focussing ( $m_1 = 1, m_2 = 24$ ), measured at depth level  $z = 1600$  m (upper part). Amplitude cross-section through the beams of Figure 14.4, measured at depth level  $z = 1600$  m (lower part).



**Fig. 14.6** Source signatures in a two reflector model with the first reflector at 500 m and the second at 800 m depth. The vertical DFS-array consists of 80 virtual source points with a vertical distance of 10 m [0-800] m.

## 14.2 Conclusion

By introducing double focussing in the source domain, horizontally as well as vertically, the illumination can be controlled in a user-oriented fashion.

The use of DFS-arrays may open a new way of looking at the migration process of sparse data in complex subsurfaces (both P and S). It may also offer a new opportunity to better utilize multipath illumination.

## 14.3 References

- Berkhout, A. J., 1997, Pushing the limits of seismic imaging, part I: Prestack migration in terms of double dynamic focusing: *Geophysics*, **62**, 937–953.
- Thorbecke, J. W., 1997, Common focus point technology: Ph.D. thesis, Delft University of Technology.