

SCATTERING BY A STRIP
IN A HOMOGENEOUS MEDIUM

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Ik ben altijd ontsteld wanneer ik iets voltooi. Ik schrik en wordt door verdriet overmand. Mijn volmaaktheidsinstinct zou me moeten beletten iets te voltooiën; het zou me zelfs moeten beletten ergens aan te beginnen. Maar ik ben verstrooid en doe het toch, met als resultaat een produkt dat bij mij niet voortkomt uit de wil, maar uit de afwezigheid ervan. Ik begin omdat ik geen kracht heb om te denken; ik voltooi omdat mijn ziel de kracht mist om eerder op te houden. Dit boek is mijn lafheid.

Fernando Pessoa, *Het Boek Der Rusteloosheid*

Preface

Two years ago, in December 1988, I was ready to start with the last few steps of my study, but I couldn't find any motivation to start with it. I was looking for something I would never find; I was asking questions which I couldn't answer. I considered stopping to study, but I couldn't find any reason why I should stop, neither could I find any reason to finish it. For some reasons, of which I don't care anymore of not knowing them, my last steps of the study came to an end.

This thesis represents the results of my final project at the Department of Mining and Petroleum Engineering, section Applied Geophysics, at the Delft University of Technology. The accompany of this project was done by J.T. Fokkema and P.M. van den Berg, which I would like to thank for their support and understanding. I also would like to thank prof. A.M. Ziolkowski for reading this report and making many helpful comments and suggestions and, everybody who was at the 'Geophysics room', to whom I could ask everything, for helping me with all aspects of my final project and for trying to make me feel at home.

The program I have written, which is based on this report, will be used to produce a reference data set for a new theory about the removal of surface related wave phenomena in the marine case which is developed by J.T. Fokkema and P.M. van den Berg.

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